The Conical Pendulum: The Tethered Airplane

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Abstract

The introductory physics lab curriculum usually has one experiment on uniform circular motion. Physics departments typically have several variable-speed rotators in storage that, if they work, no longer work well. Replacing these rotators with new ones is costly, especially when they are only used once a year. This work describes how an inexpensive (≈$10) tethered airplane powered by a small electric motor can be used to study uniform circular motion. The airplane is easy to see and entertaining to watch.

For a given string length and airspeed, a tethered airplane quickly finds a stable, horizontal, circular orbit. Using a Digital Video (DV) camcorder, VideoPoint Capture, QuickTime Player, meter sticks, and a stopwatch, data on the airplane’s motion was obtained. The length of the string was varied from 120 cm to 340 cm while the airspeed ranged from 200 cm/s to

480 cm/s. For each string length and airspeed, the period of the orbit and the diameter of the path were carefully measured. Theoretical values of path radii were also calculated using Newton's 2nd Law. The agreement between experiment and theory was usually better than 2%.

The Conical Pendulum

In theory, the conical pendulum[1-3] consists of a bob of mass *m* at the end of a string of length *L* moving with constant speed *v* in a horizontal circle of radius *r*. See Fig. 1. The bob's acceleration results from two forces: the tension $\mathbf{F_T}$ from the string which points upward toward the pivot and the weight m$\mathbf{g}$ which points downward. Along the radial and vertical directions Newton's 2nd law yields

$$\mathrm{F_T}\sin\theta + 0 = m\frac{v^2}{r}$$
$$\mathrm{F_T}\cos\theta - mg = 0,$$

respectively, where θ is the angle between the string and the vertical. Using

$$v = \frac{2\pi r}{T} \quad (1)$$

where *T* is the period of the orbit and

$$\tan\theta = \frac{r}{\sqrt{L^2 - r^2}}$$

it can be shown that the radius *r* of the orbit relates to the string length and the period as

$$r = \sqrt{L^2 - \left(\frac{gT^2}{4\pi^2}\right)^2} \quad (2)$$

In practice, a small tangential force is required to overcome air drag. In this experiment, that force is provided by a propeller driven by a small DC motor.

The Tethered Airplane

The idea of using a tethered airplane to study uniform circular motion is an obvious one.[4-5] This work describes how we have used it. The airplane used in this experiment is pictured in

Fig. 2. It is a 1/40th scale toy Cessna 210 made in China by Novatoy. Its wingspan is 9 in, its length is 8½ in, and its weight is 4.8 ozs (with two AA batteries). It attaches to a nylon string that attaches to a pivot that is mounted on the ceiling.

The quantity L mentioned above is in fact a little longer than the length of the string. It is the distance between the pivot and the center of mass (CM) of the airplane. The CM can be determined by hanging the airplane from various points using a string, as in Fig. 3, and visually projecting a line along this string. Faint lines that intersect at the CM can be etched into the body of the airplane.

The period of the orbit T was obtained by timing 10 periods with a stopwatch. See Fig. 4. Several such measurements of $10T$ were made and an average was taken provided there was agreement to within three digits.

The most difficult measurement to make was the path diameter $2r$ because the airplane is moving fast--200 cm/s to 480 cm/s. The path diameter, as well as the string length, were usually greater than two meters--the length of our longest meter sticks. This made it necessary to tape meter sticks together. For $2r$, the meter stick was positioned horizontally along a line just below the center of the circular orbit. The location of this center was clearly indicated with a bob hanging from the pivot above it as in Fig. 5. Using a DV camcorder the airplane was filmed flying just over the meter stick at either end. See Fig. 6. Several passes were filmed and the video was recorded to the hard drive of a computer in the QuickTime (.MOV) format using VideoPoint Capture set up for 60 frames per second. Afterwards, the movies were analyzed frame-by-frame using QuickTime Player to determine the two positions along the meter stick over which the CM passed. The orbits were quite stable with the airplane flying over the same two positions repeatedly. Assessing the radial positions of the CM required careful observation of the two movies and was usually made by at least two people. The diameter $2r$ was the difference between these two positions.

The reader might wonder why QuickTime Player was used instead of video analysis programs such as VideoPoint or VideoGraph. In this experiment we wanted to have large orbits that took up much of the room. In fact our longest string length exceeded the height of the room. These two programs, however, limit the size of the orbit. They require the camcorder to point directly upward so the line-of-sight is perpendicular to the plane of the motion. To work well, the entire orbit needs to be video captured. Even with our camcorder's widest angle the orbital diameter was limited to 120 cm or less--this with the camcorder on the floor, 320 cm below the pivot. In principle, video capturing just a quarter of the orbit could yield *r* and *T* from fits to x = x(t) and y = y(t) data. Our first attempts at doing this with a large orbit, however, were discouraging.

In order to obtain many data points, we also needed to vary the airspeed. The airplane was not designed for this so we did a number of things. (1) We alternated between using one and using two batteries. In this model of airplane the batteries are in parallel. The batteries were located very near the CM so the effect of adding or removing one of them was ignored. (2) We tried different combinations of weaker and stronger batteries. We rated all of our batteries by how much current they drew from a 10-Ω resistor. (3) We attached post-its to the main wing to increase drag. (4) We removed the main wing to reduce drag as in Fig. 7. This wing provides no lift and its contribution to the CM was negligible. Using various combinations of the above we were able to vary the airspeed between 200 cm/s and 480 cm/s.

Results

The 22 data points are listed in Table I. Measurements were taken for five string length's varying from 120 cm to 340 cm. For each string length, measurements of *T* and *2r* were taken for a range of airspeeds. Airspeeds were calculated using Eqn. (1) and theoretical radii were calculated using Eqn. (2). We used a value of $g = 980.2\,m/s^2$ for the acceleration due to gravity corresponding to our latitude of 40°N. The data are also presented graphically in Fig. 8 alongside the corresponding theoretical curves. The experimental discrepancies were generally less than 2%.

Application

The procedure described above for measuring *r* is accurate but it is too involved for a two-hour introductory lab. The following modifications are suggested.

(1) Adjust the string length so that the airplane is flying just over the heads of two students at opposite ends of the orbit. The students could be standing on the floor or standing on a stool. They could also be lying on lab benches or lying on the floor. Once positioned, they can hold a meter stick just above their heads and watch the airplane fly directly over. Viewed from so close, it will not be much more than a blur. Students can still record the point over which the CM is passing. It is recommended that the main wing be removed because it causes the blur to be very wide. It helps that the airplane has a small light on its belly. In fact, it is easy to see the path of this light with the ceiling lights dimmed. A third student

can check that the meter stick is lined up radially--passing under the central bob--as described earlier. This student can also operate the stopwatch.

(2) Sargent-Welch sells a tethered airplane for $50. Though more expensive, it comes rigged with a spring scale which reads the string tension $\boldsymbol{F_T}$ while the airplane is moving.[6] Students can easily compare it with

$$F_T = \frac{4\pi^2 mL}{T^2}$$

The spring scale circumvents the need to measure *r* which can be complicated and time-consuming.

(3) The Student Force Sensor from Vernier Software has also been used with the conical pendulum.[7] With the Force Sensor students can obtain the radial component of the string tension $F_T^{(r)}$ and compare it with

$$F_T^{(r)} = m\sqrt{\left(\frac{4\pi^2 L}{T^2}\right)^2 - g^2}$$

(4) Film the motion from below and capture to the hard drive as mentioned earlier. The camcorder needs to be pointing upward and a meter stick or some object of known length must be in the plane of the motion. The video can then be analyzed using a program such as VideoPoint or VideoGraph. Unfortunately, the size of the orbit is limited by the camera angle and the height of the ceiling.

It is also recommended not to bother with varying the airspeed because it is too time-consuming. A wide enough range of orbits is possible by simply varying *L*. Consider also having the students calculate *g* using

$$g = \frac{4\pi^2 \sqrt{L^2 - r^2}}{T^2}$$

instead calculating *r*. By the time they are studying circular motion they are familiar with *g* and can make a meaningful comparison with the known value.

Conclusions

The tethered airplane described above offers an inexpensive and entertaining way to study uniform circular motion in the introductory lab. The orbits can be large and the experimental discrepancies small.

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Fig. 1. The conical pendulum.

Fig. 2. The Tethered Cessna 210 Action Model airplane by Novatoy.

Fig. 3. Determining the location of the center of mass (CM).

Fig. 4. Stopping the stopwatch.

Fig. 5. Bob marking the center of the orbit.

Fig. 6. Airplane lined up with the meter stick.

Fig. 7. Airplane (with main wing removed) lined up with the meter stick.

Table I. <See below>

Fig. 8. Comparison of experimental and theoretical orbital radii for $L = 119.2$ cm, 173.7 cm, 227.7 cm, 281.8 cm, 341.1 cm.

Table I. Measurements of T and $2r$ were taken for a range of string length's L and airspeeds v. Experimental and Theoretical r's are compared.

Length L(cm)	Period T(s)	Diameter $2r$(cm)	Airspeed v(cm/s)	Experimental Radius r(cm)	Theoretical Radius r(cm)	Experimental Discrepancy
119.2	1.984	134.2	212.6	67.1	68.3	-1.8%
119.2	1.857	164.0	277.4	82.0	82.9	-1.1%
119.2	1.721	183.8	335.5	91.9	93.8	-2.0%
119.2	1.624	197.2	381.5	98.6	99.6	-1.1%
173.7	2.498	158.5	199.3	79.3	78.5	0.9%
173.7	2.476	165.5	210.0	82.8	83.7	-1.1%
173.7	2.345	212.6	284.8	106.3	107.4	-1.0%
173.7	2.215	241.7	342.8	120.9	123.8	-2.4%
173.7	1.943	288.2	466.0	144.1	146.2	-1.5%
227.7	2.873	203.6	222.6	101.8	99.2	2.6%
227.7	2.799	235.7	264.5	117.9	118.4	-0.4%
227.7	2.727	265.4	305.8	132.7	133.3	-0.5%
227.7	2.608	299.6	360.9	149.8	152.7	-1.9%
227.7	2.334	359.2	483.5	179.6	183.2	-2.0%
281.8	3.245	215.7	208.9	107.9	105.3	2.4%
281.8	3.173	262.1	259.5	131.1	130.1	0.7%
281.8	3.085	309.4	315.1	154.7	153.5	0.8%
281.8	2.917	369.3	397.7	184.7	186.5	-1.0%
281.8	2.789	407.6	459.1	203.8	205.2	-0.7%
341.1	3.517	292.8	261.5	146.4	148.4	-1.4%
341.1	3.487	328.2	295.7	164.1	158.9	3.3%
341.1	3.210	445.1	435.6	222.6	225.6	-1.4%