Previous lecture

- Demography is the study of controls of birth and death in populations
  - Populations are structured
- Useful summary statistics
  - Life table
  - e_x ; life expectancy
  - Net reproductive rate

Lecture 4

- Population Projection
- λ
- Propensity fitness

Population Projection

- Population prediction:
- Population projection:

Population Projection

- Knowing survival rates and birth rates, we can project future population sizes
- 2 expressions of survival and fertility are needed for projections:
  - 1. s_x
  - 2. F_x

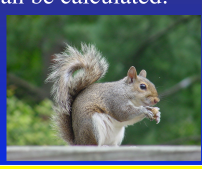
Note: In many texts, and old sample Bio 221 probs, s_x is called p_x

Population Projection

- Knowing survival rates and birth rates, we can project future population sizes
- 2 expressions of survival and fertility are needed for projections:
  - 1. s_x $(= l_{x+1}/l_x)$
  - 2. F_x $(=s_x b_{x+1})$

Population Projection

- Knowing survival probabilities (s_x), the number of individuals moving into the next age class can be calculated:



$$N_{x+1}(t+1) = s_x N_x(t)$$

Population Projection (part 2)

- To complete the accounting, newborn numbers need to be estimated using fertilities (F_x):

$$N_0(t+1) = \sum_{x=0}^{lastage} F_x N_x(t)$$

Population Projection Equations

$$N_0(t+1) = \sum_{x=0}^{lastage} F_x N_x(t)$$

$$N_{x+1}(t+1) = s_x N_x(t)$$

Sample Projection

- The projection equations are used to determine $N_{(x+1)}(t+1)$:

Table 10.6 | Population Projection Table, Squirrel Population

	Year (t)										
Age	0	1	2	3	4	5	6	7	8	9	10
0	20	27	34.1	40.71	48.21	58.37	70.31	84.8	101.86	122.88	148.06
1	10	6	8.1	10.23	12.05	14.46	17.51	21.0	25.44	30.56	36.86
2	0	5	3.0	4.05	5.1	6.03	7.23	8.7	10.50	12.72	15.28
3	0	0	3.0	1.8	2.43	3.06	3.62	4.4	5.22	6.30	7.63
4	0	0	0	1.35	0.81	1.09	1.38	1.6	1.94	2.35	2.83
5	0	0	0	0	0.33	0.20	0.27	0.35	0.40	0.49	0.59
Total $N(t)$	30	38	48.2	58.14	68.93	83.21	100.32	120.85	145.36	175.30	211.25
Lambda	λ	1.27	1.27	1.21	1.19	1.21	1.20	1.20	1.20	1.20	1.20

One ‘iteration’ of this projection (starting with year 2)

$$N_x(t) = \begin{pmatrix} 34.1 \\ 8.1 \\ 3 \\ 3 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} N_0(t+1) \\ N_1(t+1) \\ N_2(t+1) \\ N_3(t+1) \\ N_4(t+1) \end{pmatrix}$$

34.1*(0.3)

8.1*(0.5)

Etc.

Try it!

Yields…

$$N_x(t) = \begin{pmatrix} 34.1 \\ 8.1 \\ 3 \\ 3 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} N_0(t+1) \\ 10.23 \\ 4.05 \\ 1.8 \\ 1.35 \end{pmatrix}$$

34.1*(0.3)

8.1*(0.5)

Etc.

Calculate Newborns (N_0)

$$N_0(t+1) = \sum_{x=0}^{lastage} F_x N_x(t)$$

$$N_0(t+1) = (0.6*34.1) + (1.5*8.1) + (1.8*3) + (0.88*3)$$

$$N_0(t+1) = 40.65$$

Note: Your book has a bit of rounding error for this…

Total Population Size

- Just sum across age classes!

$$N_{total} = \sum_{x=0}^{x} N_x$$

Sample Projection

- The projection equations are used to determine $N_{(x+1)}(t+1)$:

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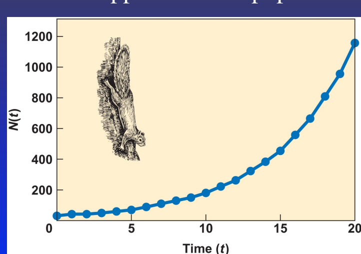
4	0	0	0	1.35	0.81	1.09	1.38	1.6	1.94	2.35	2.83
5	0	0	0	0	0.33	0.20	0.27	0.35	0.40	0.49	0.59
Total $N(t)$	30	38	48.2	58.14	68.93	83.21	100.32	120.85	145.36	175.30	211.25
~~Lambda~~	λ	1.27	1.27	1.21	1.19	1.21	1.20	1.20	1.20	1.20	1.20

N(t+1)/N(t)

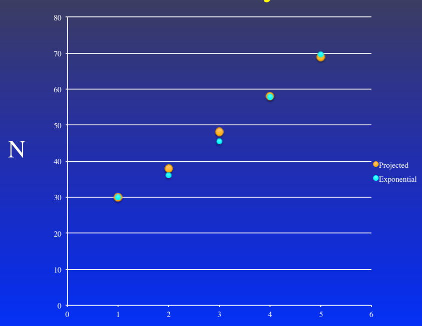
Population Projection

- What has happened to the population?



A) It has grown exponentially, B) It has grown faster than exponentially, C) It has grown slower than exponentially

First 5 years



Stable Age Distribution

- Structured populations actually grow exponentially only after reaching the stable age distribution:

Table 10.7 | Approximation of Stable Age Distribution, Squirrel Population

	Proportion in Each Age Class for Year										
Age	0	1	2	3	4	5	6	7	8	9	10
0	.67	.71	.71	.71	.69	.70	.70	.70	.70	.70	.70
1	.33	.16	.17	.17	.20	.17	.17	.18	.18	.18	.18
2		.13	.06	.07	.06	.07	.07	.07	.07	.07	.07
3			.06	.03	.03	.04	.04	.03	.03	.03	.03
4				.02	.01	.01	.01	.01	.01	.01	.01
5					.01	.01	.01	.01	.01	.01	.01

Tab. 9.7 from S&S – NOTE THIS IS CORRECT….

Clicker Question

Table 10.7 | Approximation of Stable Age Distribution, Squirrel Population

	Proportion in Each Age Class for Year										
Age	0	1	2	3	4	5	6	7	8	9	10
0	.67	.71	.71	.71	.69	.70	.70	.70	.70	.70	.70
1	.33	.16	.17	.17	.20	.17	.17	.18	.18	.18	.18
2		.13	.06	.07	.06	.07	.07	.07	.07	.07	.07
3			.06	.03	.03	.04	.04	.03	.03	.03	.03
4				.02	.01	.01	.01	.01	.01	.01	.01
5					.01	.01	.01	.01	.01	.01	.01

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In which year did the squirrel population reach S.A.D?
A. Year 3 B. Year 4 C. Year 5 D. Year 6 E. Year 7

Lambda (λ), the finite rate of increase

- At S.A.D., the population grows at the finite rate of increase (λ).

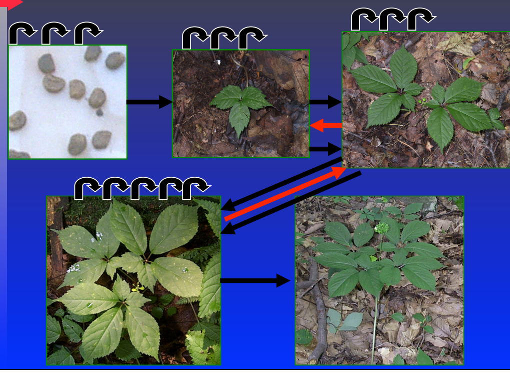
How demographers ACTUALLY do projections....

- With matrix algebra!
- λ, the finite rate of increase, is the **eigenvalue** (a complex function) of the matrix on the left in the equation

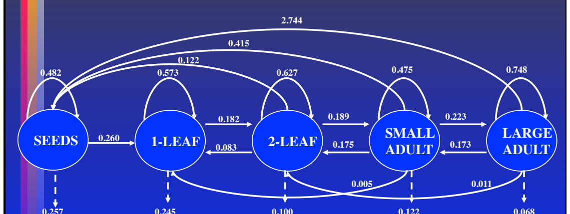
Other population projection models

- Classify individuals by size or stage: particularly important for animals or plants whose birth and death rates depend more on size than age.
- a_{ij} is the number of size i individuals (at time t +1) per size j individual (time t).

This is the size-structured 'life history' of American ginseng



Stage-based Population Projection

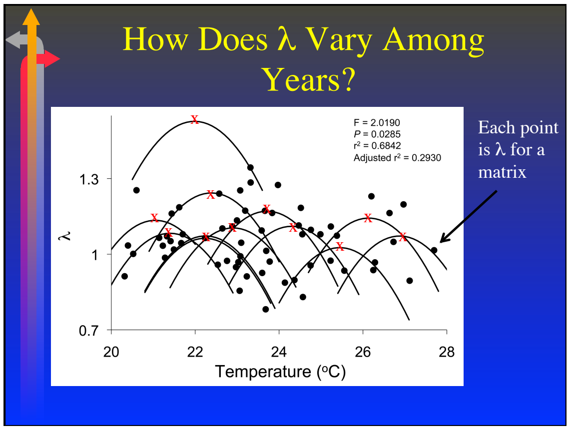


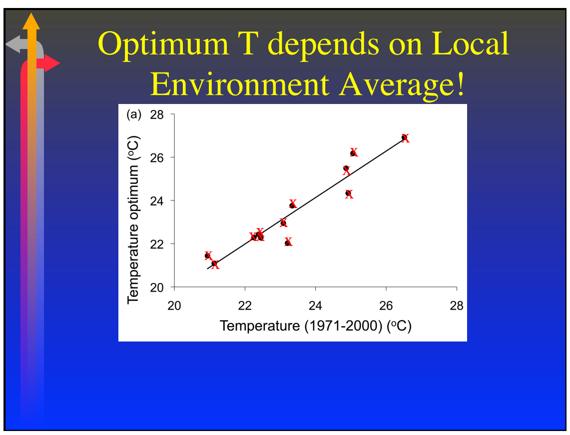
Demographic model of American ginseng

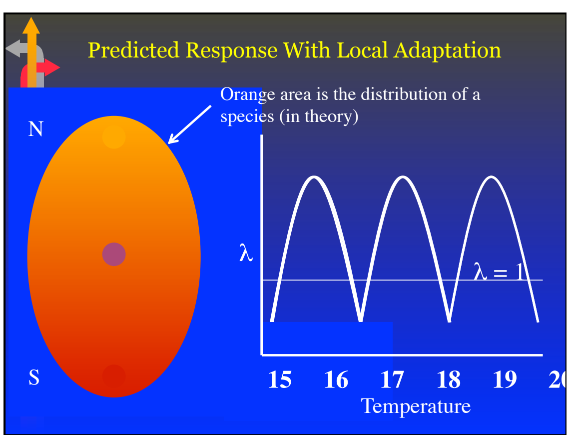
Population Projection Matrix

From: / To:	Class 1 (seeds)	Class 2 (1-leaf)	Class 3 (2-leaf)	Class 4 (sm. adult)	Class 5 (lg. adult)
Class 1	a_{11}	0	a_{13}	a_{14}	a_{15}
Class 2	a_{21}	a_{22}	a_{23}	a_{24}	0
Class 3	0	a_{32}	a_{33}	a_{34}	a_{35}
Class 4	0	0	a_{43}	a_{44}	a_{45}
Class 5	0	0	0	a_{54}	a_{55}

λ is the eigenvalue of this matrix!


How Does λ Vary Among Years?
F = 2.0190
P = 0.0285
r² = 0.6842
Adjusted r² = 0.2930
Each point is λ for a matrix
λ
1.3
1
0.7
20
22
24
26
28
Temperature (°C)


Optimum T depends on Local Environment Average!
(a)
Temperature optimum (°C)
28
26
24
22
20
20
22
24
26
28
Temperature (1971-2000) (°C)

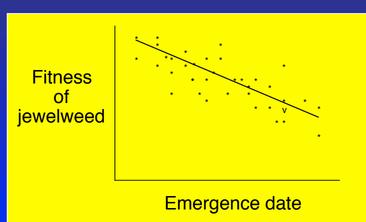

Predicted Response With Local Adaptation
Orange area is the distribution of a species (in theory)
N
S
λ
λ = 1
15
16
17
18
19
Temperature

What is fitness?

An individual has a high fitness if: A) it has the most mates B) it leaves the most offspring, C) it survives the best, D) it can lift the most weights, E) none of the above

Need for a fitness measure

- In studies of selection, the 'value' of the phenotype is judged by fitness, for example:



Propensity fitness

- Fitness is a property of the individual
- Fitness should measure the rate at which that individual's genes are propagated
- The **propensity fitness** is the expected 'population growth rate of the individual', where λ is measured for a matrix constructed for each individual in the population

Propensity fitness

- We determine an **individual's** propensity to produce a certain number of offspring at each age and to survive at each age, then fill in the traditional matrix:
- $\lambda^{(i)}$ for this matrix gives the individual's 'propensity fitness'

Fitness

- Because $\lambda^{(i)}$ is determined by the eigenvalue of the matrix $A^{(i)}$, we see that:
  - Fitness depends on the probability of survival
  - Fitness depends on the amount of reproduction
  - Fitness depends on the timing of that reproduction

What is fitness (revisited)?

An individual has a high fitness if: A) it has the most mates B) it leaves the most offspring, C) it survives the best, D) it can lift the most weights, E) none of the above

What Have We Learned So Far?

- Lecture 1 - Population statistics
  - censusing methods (including mark-recapture ($N=Mn/R$)
  - Distribution
  - dispersion ($I=V/mean$)
- Lecture 2 - Beginning population dynamics
  - Exponential growth: $dN/dt=rN$, $N(t)=N(0)e^{rt}$

What Have We Learned So Far?

- Lecture 3 - Demography (and age structure)
  - Deevey curves
  - Life table and derivatives
    - Life expectancy
    - Net reproductive rate
- Lecture 4 – Demography & Fitness
  - Population Projection
  - Propensity Fitness

Summary

- The life table yields data useful for demographic **projections**
- The projection equations allow us to:
  - estimate **finite rates of increase** (λ)
  - determine the **stable age distribution**
- If the matrix represents individual probabilities of surviving and reproducing at each age (rather than population averages), then λ = fitness