Demography is the study of controls of birth and death in populations
- Populations are structured
- Useful summary statistics
  - Life table
  - $e_x$; life expectancy
  - Net reproductive rate

Population Projection
- Population prediction
- Population projection

Lecture 4
- Population Projection
- $\lambda$
- Propensity fitness
Knowing survival rates and birth rates, we can project future population sizes. Two expressions of survival and fertility are needed for projections:

1. \( s_x \)
2. \( F_x \)

Note: In many texts, and old sample Bio 221 probs, \( s_x \) is called \( p_x \).

Knowing survival probabilities \( s_x \), the number of individuals moving into the next age class can be calculated:

\[
N_{x+1}(t+1) = s_x N_x(t)
\]
Population Projection (part 2)

To complete the accounting, newborn numbers need to be estimated using fertilities ($F_x$):

$$N_0(t+1) = \sum_{x=0}^{lastage} F_x N_x(t)$$

Population Projection Equations

$$N_0(t+1) = \sum_{x=0}^{lastage} F_x N_x(t)$$

$$N_{x+1}(t+1) = s_x N_x(t)$$

Sample Projection

The projection equations are used to determine $N_{(x+1)(t+1)}$:
One ‘iteration’ of this projection (starting with year 2)

\[
N_3(t) = \begin{pmatrix}
34.1 \\
8.1 \\
3 \\
3 \\
0
\end{pmatrix} \times \begin{pmatrix}
N_0(t+1) \\
N_1(t+1) \\
N_2(t+1) \\
N_3(t+1) \\
N_4(t+1)
\end{pmatrix}
\]

Yields…

\[
N_4(t) = \begin{pmatrix}
34.1 \\
8.1 \\
3 \\
3 \\
0
\end{pmatrix} \times \begin{pmatrix}
N_0(t+1) \\
N_1(t+1) \\
N_2(t+1) \\
N_3(t+1) \\
N_4(t+1)
\end{pmatrix} = \begin{pmatrix}
10.23 \\
4.05 \\
1.8 \\
1.35
\end{pmatrix}
\]

Calculate Newborns (N₀)

\[
N_0(t+1) = \sum_{i=0}^{\infty} F_i N_i(t)
\]

\[
N_0(t+1) = (0.6 \times 34.1) + (1.5 \times 8.1) + (1.8 \times 3) + (0.88 \times 3)
\]

\[
N_0(t+1) = 40.65
\]

Note: Your book has a bit of rounding error for this…
Total Population Size

- Just sum across age classes!

\[ N_{\text{total}} = \sum_{x=0}^{x} N_x \]

Sample Projection

- The projection equations are used to determine \( N_{(x+1)}(t+1) \):

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<th>Year (t)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<td>27</td>
<td>14.1</td>
<td>8.71</td>
<td>4.66</td>
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<td>1.83</td>
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<td>0</td>
</tr>
</tbody>
</table>

\[ \frac{N(t+1)}{N(t)} \]

Population Projection

- What has happened to the population?

A) It has grown exponentially, B) It has grown faster than exponentially, C) It has grown slower than exponentially
First 5 years

Stable Age Distribution

Structured populations actually grow exponentially only after reaching the stable age distribution:

Tab. 9.7 from S&S – NOTE THIS IS CORRECT….

Clicker Question

In which year did the squirrel population reach S.A.D?
A. Year 3   B. Year 4   C. Year 5   D. Year 6   E. Year 7
Lambda ($\lambda$), the finite rate of increase

- At S.A.D., the population grows at the finite rate of increase ($\lambda$).

How demographers ACTUALLY do projections....

- With matrix algebra!

- $\lambda$, the finite rate of increase, is the eigenvalue (a complex function) of the matrix on the left in the equation.

Other population projection models

- Classify individuals by size or stage; particularly important for animals or plants whose birth and death rates depend more on size than age.

- $a_{ij}$ is the number of size $i$ individuals (at time $t +1$) per size $j$ individual (time $t$).
This is the size-structured "life history" of American ginseng

**Stage-based Population Projection**

Demographic model of American ginseng

**Population Projection Matrix**

<table>
<thead>
<tr>
<th>From</th>
<th>Class 1 (seeds)</th>
<th>Class 2 (1-leaf)</th>
<th>Class 3 (2-leaf)</th>
<th>Class 4 (sm. adult)</th>
<th>Class 5 (lg. adult)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>a₁₁</td>
<td>0</td>
<td>a₁₃</td>
<td>0</td>
<td>a₁₅</td>
</tr>
<tr>
<td>Class 2</td>
<td>a₂₁</td>
<td>a₂₂</td>
<td>a₂₃</td>
<td>a₂₄</td>
<td>0</td>
</tr>
<tr>
<td>Class 3</td>
<td>0</td>
<td>a₃₂</td>
<td>a₃₃</td>
<td>a₃₄</td>
<td>a₃₅</td>
</tr>
<tr>
<td>Class 4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>a₄₄</td>
<td>a₄₅</td>
</tr>
<tr>
<td>Class 5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>a₅₅</td>
</tr>
</tbody>
</table>

λ is the eigenvalue of this matrix!
How Does $\lambda$ Vary Among Years?

Each point is $\lambda$ for a matrix

Optimum T depends on Local Environment Average!

Predicted Response With Local Adaptation

Orange area is the distribution of a species (in theory)
What is fitness?

An individual has a high fitness if: A) it has the most mates, B) it leaves the most offspring, C) it survives the best, D) it can lift the most weights, E) none of the above

Need for a fitness measure

In studies of selection, the ‘value’ of the phenotype is judged by fitness, for example:

Propensity fitness

Fitness is a property of the individual
Fitness should measure the rate at which that individual’s genes are propagated
The propensity fitness is the expected ‘population growth rate of the individual’, where $\lambda$ is measured for a matrix constructed for each individual in the population
We determine an individual’s propensity to produce a certain number of offspring at each age and to survive at each age, then fill in the traditional matrix:

\[ \lambda^{(i)} \]

for this matrix gives the individual’s ‘propensity fitness’

Fitness

Because \( \lambda^{(0)} \) is determined by the eigenvalue of the matrix \( A^{(0)} \), we see that:

- Fitness depends on the probability of survival
- Fitness depends on the amount of reproduction
- Fitness depends on the timing of that reproduction

What is fitness (revisited)?

An individual has a high fitness if: A) it has the most mates B) it leaves the most offspring, C) it survives the best, D) it can lift the most weights, E) none of the above
What Have We Learned So Far?

Lecture 1 - Population statistics
- Censusing methods (including mark-recapture \( N = Mn/R \))
- Distribution
- Dispersion \( (I = V/\text{mean}) \)

Lecture 2 - Beginning population dynamics
- Exponential growth: \( \frac{dN}{dt} = rN \), \( N(t) = N(0)e^{rt} \)

What Have We Learned So Far?

Lecture 3 - Demography (and age structure)
- Deevey curves
- Life table and derivatives
- Life expectancy
- Net reproductive rate

Lecture 4 – Demography & Fitness
- Population Projection
- Propensity Fitness

Summary
- The life table yields data useful for demographic projections
- The projection equations allow us to:
  - Estimate finite rates of increase \( (\lambda) \)
  - Determine the stable age distribution
- If the matrix represents individual probabilities of surviving and reproducing at each age (rather than population averages), then \( \lambda = \text{fitness} \)