## Previous lecture

- Demography is the study of controls of birth and death in populations
  - Populations are structured
- Useful summary statistics
  - Life table
  - $e_x$ ; life expectancy
  - Net reproductive rate

## Lecture 4

- Population Projection
- λ
- Propensity fitness

## Population Projection

- Population prediction:
- Population projection:

## Population Projection

- Knowing survival rates and birth rates, we can project future population sizes
- 2 expressions of survival and fertility are needed for projections:
  - 1. $s_x$
  - 2. $F_x$

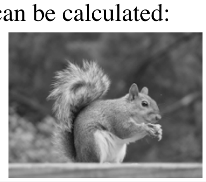
Note: In many texts, and old sample Bio 221 probs, $s_x$ is called $p_x$

## Population Projection

- Knowing survival rates and birth rates, we can project future population sizes
- 2 expressions of survival and fertility are needed for projections:
  - 1. $s_x$ (= $l_{x+1}/l_x$)
  - 2. $F_x$ (=$s_x b_{x+1}$)

## Population Projection

- Knowing survival probabilities ($s_x$), the number of individuals moving into the next age class can be calculated:



$$N_{x+1}(t+1) = s_x N_x(t)$$

## Population Projection (part 2)

- To complete the accounting, newborn numbers need to be estimated using fertilities ($F_x$):

$$N_0(t+1) = \sum_{x=0}^{lastage} F_x N_x(t)$$

## Population Projection Equations

$$N_0(t+1) = \sum_{x=0}^{lastage} F_x N_x(t)$$

$$N_{x+1}(t+1) = s_x N_x(t)$$

## Sample Projection

- The projection equations are used to determine $N_{(x+1)}(t+1)$:

Table 10.6 | Population Projection Table, Squirrel Population

| | Year ($t$) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Age | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 0 | 20 | 27 | 34.1 | 40.71 | 48.21 | 58.37 | 70.31 | 84.8 | 101.86 | 122.88 | 148.06 |
| 1 | 10 | 6 | 8.1 | 10.23 | 12.05 | 14.46 | 17.51 | 21.0 | 25.44 | 30.56 | 36.86 |
| 2 | 0 | 5 | 3.0 | 4.05 | 5.1 | 6.03 | 7.23 | 8.7 | 10.50 | 12.72 | 15.28 |
| 3 | 0 | 0 | 3.0 | 1.8 | 2.43 | 3.06 | 3.62 | 4.4 | 5.22 | 6.30 | 7.63 |
| 4 | 0 | 0 | 0 | 1.35 | 0.81 | 1.09 | 1.38 | 1.6 | 1.94 | 2.35 | 2.83 |
| 5 | 0 | 0 | 0 | 0 | 0.33 | 0.20 | 0.27 | 0.35 | 0.40 | 0.49 | 0.59 |
| Total $N(t)$ | 30 | 38 | 48.2 | 58.14 | 68.93 | 83.21 | 100.32 | 120.85 | 145.36 | 175.30 | 211.25 |
| Lambda | $\lambda$ | 1.27 | 1.27 | 1.21 | 1.19 | 1.21 | 1.20 | 1.20 | 1.20 | 1.20 | 1.20 |

## One ‘iteration’ of this projection (starting with year 2)

$$N_x(t) = \begin{pmatrix} 34.1 \\ 8.1 \\ 3 \\ 3 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} N_0(t+1) \\ N_1(t+1) \\ N_2(t+1) \\ N_3(t+1) \\ N_4(t+1) \end{pmatrix}$$

34.1*(0.3)
8.1*(0.5)
Etc.

Try it!

## Yields…

$$N_x(t) = \begin{pmatrix} 34.1 \\ 8.1 \\ 3 \\ 3 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} N_0(t+1) \\ 10.23 \\ 4.05 \\ 1.8 \\ 1.35 \end{pmatrix}$$

34.1*(0.3)
8.1*(0.5)
Etc.

## Calculate Newborns ($N_0$)

$$N_0(t+1) = \sum_{x=0}^{lastage} F_x N_x(t)$$

$$N_0(t+1) = (0.6*34.1) + (1.5*8.1) + (1.8*3) + (0.88*3)$$

$$N_0(t+1) = 40.65$$

Note: Your book has a bit of rounding error for this…

# Total Population Size

- Just sum across age classes!

$$N_{total} = \sum_{x=0}^{x} N_x$$

# Sample Projection

- The projection equations are used to determine $N_{(x+1)}(t+1)$:

Table 10.6 | **Population Projection Table, Squirrel Population**

| | Year ($t$) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Age | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 0 | 20 | 27 | 34.1 | 40.71 | 48.21 | 58.37 | 70.31 | 84.8 | 101.86 | 122.88 | 148.06 |
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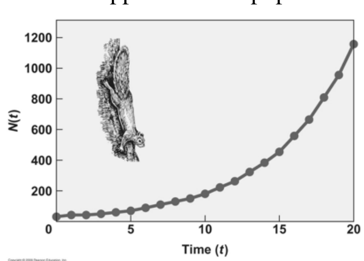
| 5 | 0 | 0 | 0 | 0 | 0.33 | 0.20 | 0.27 | 0.35 | 0.40 | 0.49 | 0.59 |
| Total $N(t)$ | 30 | 38 | 48.2 | 58.14 | 68.93 | 83.21 | 100.32 | 120.85 | 145.36 | 175.30 | 211.25 |
| lambda | λ | 1.27 | 1.27 | 1.21 | 1.19 | 1.21 | 1.20 | 1.20 | 1.20 | 1.20 | 1.20 |



N(t+1)/N(t)

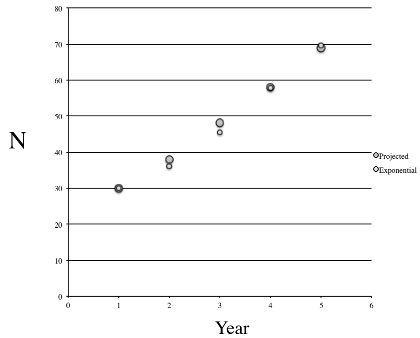
# Population Projection

- What has happened to the population?




A) It has grown exponentially, B) It has grown faster than exponentially, C) It has grown slower than exponentially

# First 5 years




# Stable Age Distribution

- Structured populations actually grow exponentially only after reaching the stable age distribution:

Table 10.7 | Approximation of Stable Age Distribution, Squirrel Population

| | Proportion in Each Age Class for Year | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Age | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 0 | .67 | .71 | .71 | .71 | .69 | .70 | .70 | .70 | .70 | .70 | .70 |
| 1 | .33 | .16 | .17 | .17 | .20 | .17 | .17 | .18 | .18 | .18 | .18 |
| 2 | | .13 | .06 | .07 | .06 | .07 | .07 | .07 | .07 | .07 | .07 |
| 3 | | | .06 | .03 | .03 | .04 | .04 | .03 | .03 | .03 | .03 |
| 4 | | | | .02 | .01 | .01 | .01 | .01 | .01 | .01 | .01 |
| 5 | | | | | .01 | .01 | .01 | .01 | .01 | .01 | .01 |



Tab. 9.7 from S&S – NOTE THIS IS CORRECT….

# Clicker Question

Table 10.7 | Approximation of Stable Age Distribution, Squirrel Population

| | Proportion in Each Age Class for Year | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Age | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 0 | .67 | .71 | .71 | .71 | .69 | .70 | .70 | .70 | .70 | .70 | .70 |
| 1 | .33 | .16 | .17 | .17 | .20 | .17 | .17 | .18 | .18 | .18 | .18 |
| 2 | | .13 | .06 | .07 | .06 | .07 | .07 | .07 | .07 | .07 | .07 |
| 3 | | | .06 | .03 | .03 | .04 | .04 | .03 | .03 | .03 | .03 |
| 4 | | | | .02 | .01 | .01 | .01 | .01 | .01 | .01 | .01 |
| 5 | | | | | .01 | .01 | .01 | .01 | .01 | .01 | .01 |

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In which year did the squirrel population reach S.A.D?
A. Year 3 B. Year 4 C. Year 5 D. Year 6 E. Year 7

## Lambda (λ), the finite rate of increase

- At S.A.D., the population grows at the finite rate of increase (λ).

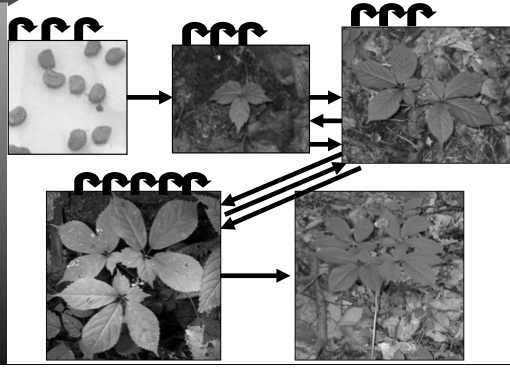
## How demographers ACTUALLY do projections....


- With matrix algebra!
- λ, the finite rate of increase, is the **eigenvalue** (a complex function) of the matrix on the left in the equation

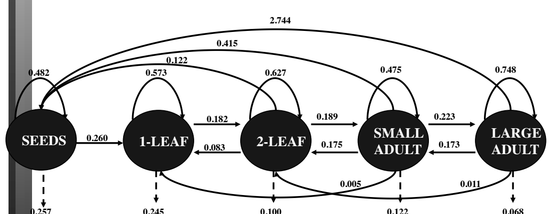
## Other population projection models

- Classify individuals by size or stage: particularly important for animals or plants whose birth and death rates depend more on size than age.
- $a_{ij}$ is the number of size i individuals (at time t +1) per size j individual (time t).

This is the size-structured ‘life history’ of American ginseng



# Stage-based Population Projection



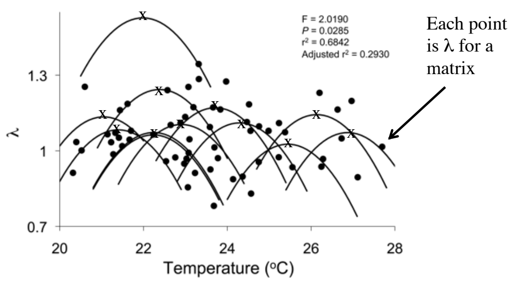

Demographic model of American ginseng

# Population Projection Matrix

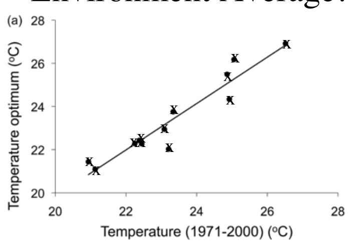
| To: / From: | Class 1 (seeds) | Class 2 (1-leaf) | Class 3 (2-leaf) | Class 4 (sm. adult) | Class 5 (lg. adult) |
|---|---|---|---|---|---|
| Class 1 | $a_{11}$ | 0 | $a_{13}$ | $a_{14}$ | $a_{15}$ |
| Class 2 | $a_{21}$ | $a_{22}$ | $a_{23}$ | $a_{24}$ | 0 |
| Class 3 | 0 | $a_{32}$ | $a_{33}$ | $a_{34}$ | $a_{35}$ |
| Class 4 | 0 | 0 | $a_{43}$ | $a_{44}$ | $a_{45}$ |
| Class 5 | 0 | 0 | 0 | $a_{54}$ | $a_{55}$ |

$\lambda$ is the eigenvalue of this matrix!

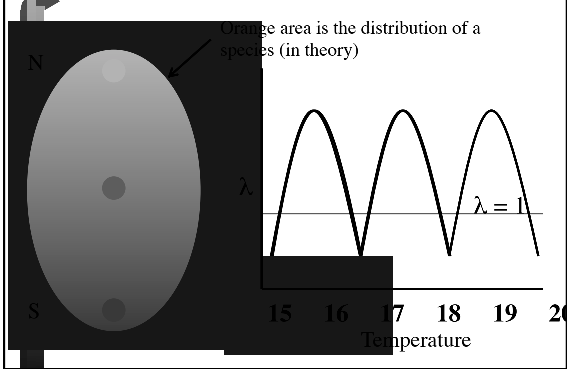
# How Does λ Vary Among Years?




# Optimum T depends on Local Environment Average!




## Predicted Response With Local Adaptation

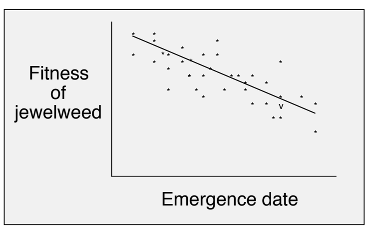


## What is fitness?


An individual has a high fitness if: A) it has the most mates B) it leaves the most offspring, C) it survives the best, D) it can lift the most weights, E) none of the above

## Need for a fitness measure

- In studies of selection, the value of the phenotype is judged by fitness, for example:




## Propensity fitness

- Fitness is a property of the individual
- Fitness should measure the rate at which that individual s genes are propagated
- The **propensity fitness** is the expected ‘population growth rate of the individual’, where λ is measured for a matrix constructed for each individual in the population

## Propensity fitness

- We determine an **individual's** propensity to produce a certain number of offspring at each age and to survive at each age, then fill in the traditional matrix:
- $\lambda^{(i)}$ for this matrix gives the individual's 'propensity fitness'

## Fitness

- Because $\lambda^{(i)}$ is determined by the eigenvalue of the matrix $A^{(i)}$, we see that:
  - Fitness depends on the probability of survival
  - Fitness depends on the amount of reproduction
  - Fitness depends on the timing of that reproduction

## What is fitness (revisited)?


An individual has a high fitness if: A) it has the most mates B) it leaves the most offspring, C) it survives the best, D) it can lift the most weights, E) none of the above

## What Have We Learned So Far?

- Lecture 1 - Population statistics
  - censusing methods (including mark-recapture ($N=Mn/R$)
  - Distribution
  - dispersion ($I=V/mean$)
- Lecture 2 - Beginning population dynamics
  - Exponential growth: $dN/dt=rN$, $N(t)=N(0)e^{rt}$

## What Have We Learned So Far?

- Lecture 3 - Demography (and age structure)
  - Deevey curves
  - Life table and derivatives
    - Life expectancy
    - Net reproductive rate
- Lecture 4 – Demography & Fitness
  - Population Projection
  - Propensity Fitness

## Summary

- The life table yields data useful for demographic **projections**
- The projection equations allow us to:
  - estimate **finite rates of increase** ($\lambda$)
  - determine the **stable age distribution**
- If the matrix represents individual probabilities of surviving and reproducing at each age (rather than population averages), then $\lambda$ = fitness