

Lecture 16

Model development  
Six Cases: 1 gene, 2 allele model  
With exciting real-world examples!  
Article 15

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Model of Natural Selection

→ Allele number passed on is modified by survival and fertility. Assume 'big bang' life history, then

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Model of selection

→ Likewise,

→ We now have the numerator, and continuing in like fashion, we derive the denominator (=all alleles)

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## Model of Selection

- Yielding,

- To simplify, factor 2 and  $N_t$  from the numerator and denominator, and let  $l \bullet m = W$  (absolute fitness). We will use relative fitness ( $w$ , rather than  $W$ ), where

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## Model of Selection

- The selection equation can now be 'iterated' to project future allele frequency change, given initial  $p$ , and fitnesses associated with each genotype.

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## Iteration of the Selection Model

- Selection model is deterministic (unlike drift), predicting specific allele frequency changes

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### The Selection Equation

$$p_{t+1} = \frac{w_1 p_t^2 + w_1 2 p_t q_t}{w_1 p_t^2 + 2 w_1 2 p_t q_t + w_2 2 q_t^2}$$

Case	Genotype	Phenotype	Fitness
1	A <sub>1</sub> A <sub>1</sub>	Purple	1
	A <sub>1</sub> A <sub>2</sub>	Pink	1
	A <sub>2</sub> A <sub>2</sub>	White	1

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### The Selection Equation

$$p_{t+1} = \frac{w_1 p_t^2 + w_1 2 p_t q_t}{w_1 p_t^2 + 2 w_1 2 p_t q_t + w_2 2 q_t^2}$$

Case	Genotype	Phenotype	Fitness
1	★ A <sub>1</sub> A <sub>1</sub>	Purple	1
	★ A <sub>1</sub> A <sub>2</sub>	Pink*	1
	★ A <sub>2</sub> A <sub>2</sub>	White	1

\*Note; with equal fitnesses, the phenotype of the heterozygote does not matter. The results will be the same

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### Make a prediction!

Case	Genotype	Phenotype	Fitness
1	★ A <sub>1</sub> A <sub>1</sub>	Purple	1
	★ A <sub>1</sub> A <sub>2</sub>	Pink	1
	★ A <sub>2</sub> A <sub>2</sub>	White	1

(A) p will decrease, resulting in fixation of A<sub>2</sub>  
 (B) p will decrease, and approach 0 asymptotically  
 (C) p will remain the same  
 (D) p will increase, approaching 1 asymptotically  
 (E) p will increase, resulting in fixation of A<sub>1</sub>

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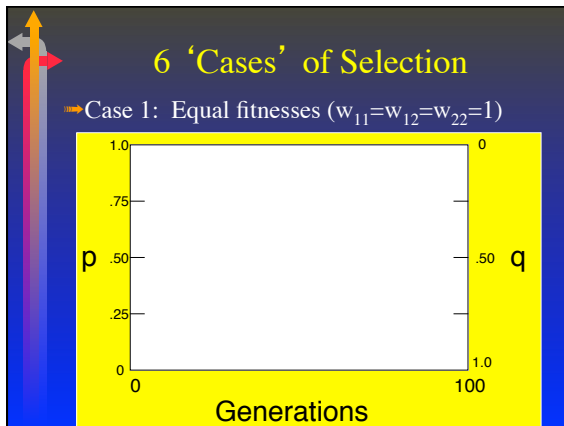
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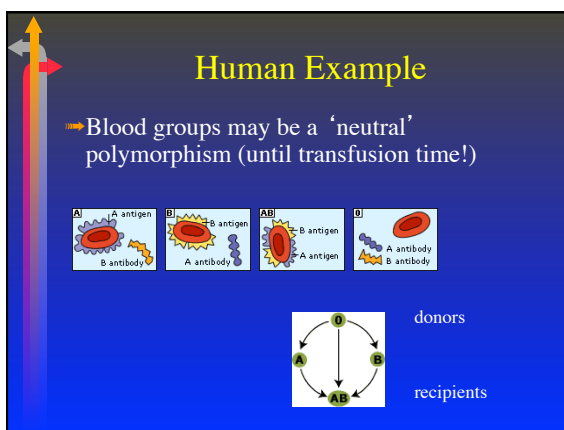
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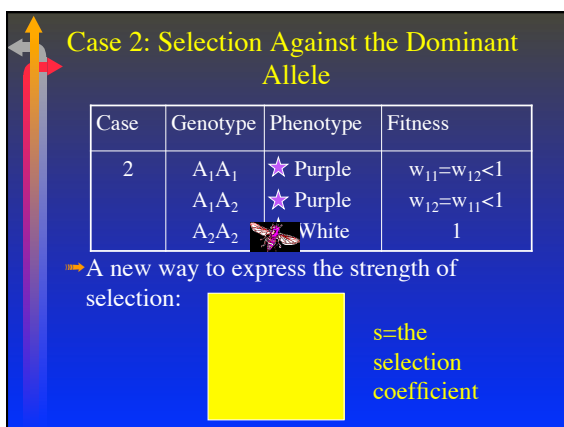
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### Case 2: Selection against dominant

#### What will happen?

Case	Genotype	Phenotype	Fitness
2	$A_1A_1$	★ Purple	$w_{11}=w_{12}<1$
	$A_1A_2$	★ Purple	$w_{12}=w_{11}<1$
	$A_2A_2$	White 	1

(A)  $p$  will decrease, resulting in fixation of  $A_2$   
 (B)  $p$  will decrease, and approach 0 asymptotically  
 (C)  $p$  will reach an equilibrium between 0 and 1  
 (D)  $p$  will increase, approaching 1 asymptotically  
 (E)  $p$  will increase, resulting in fixation of  $A_1$

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### How do we determine the outcome?

- Which allele will increase?
- How 'exposed' is the allele that is declining?

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### Human Example

- Huntington's Disease - dominant gene causes gradual disruption of nerve function
- Why is it still around? Mutation. Effects occur after most reproduction.
- High frequency near Lake Maracaibo; founder effect traced to one early immigrant who bore 10 children!
- [http://evolution.berkeley.edu/evolibrary/article/0\\_0\\_0/medicine\\_05](http://evolution.berkeley.edu/evolibrary/article/0_0_0/medicine_05)

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### Case 3 - Selection Against the Recessive

Case	Genotype	Phenotype	Fitness	Selection coefficient
3	$A_1A_1$	 Purple	1	0
	$A_1A_2$	 Purple	1	0
	$A_2A_2$	★ White	$w_{22} < 1$	$s_{22}$

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### Case 3 – Selection against the recessive. Make a prediction!

Case	Genotype	Phenotype	Fitness	Selection coefficient
3	$A_1A_1$	 Purple	1	0
	$A_1A_2$	 Purple	1	0
	$A_2A_2$	★ White	$w_{22} < 1$	$s_{22}$

(A)  $p$  will decrease, resulting in fixation of  $A_2$   
 (B)  $p$  will decrease, and approach 0 asymptotically  
 (C)  $p$  will reach an equilibrium between 0 and 1  
 (D)  $p$  will increase, approaching 1 asymptotically  
 (E)  $p$  will increase, resulting in fixation of  $A_1$

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### Human Example

- Cystic fibrosis (recessive allele in CF gene); most common lethal genetic disease of European and European derived populations!
- Blockages in sweat and mucus glands; leads to respiratory failure, etc.
- May be still around because (1) recessive lethals are difficult to get rid of when in low proportions, and (2) in the past, the homozygotes may have had an advantage in defense against tuberculosis!
- Read more: [http://www.nhlbi.nih.gov/health/dci/Diseases/cf/cf\\_what.html](http://www.nhlbi.nih.gov/health/dci/Diseases/cf/cf_what.html)

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### Case 4: No dominance - linear arrangement of fitnesses

Case	Genotype	Phenotype	Fitness	Selection coefficient
4	$A_1A_1$	★ Purple	$w_{11} < w_{12}$	$s_{11} > s_{12}$
	$A_1A_2$	Pink	$w_{12} < 1$	$s_{12}$
	$A_2A_2$	White	1	0

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### Case 4 - No dominance

#### Make a prediction!

Case	Genotype	Phenotype	Fitness	Selection coefficient
4	$A_1A_1$	★ Purple	$w_{11} < w_{12}$	$s_{11} > s_{12}$
	$A_1A_2$	Pink	$w_{12} < 1$	$s_{12}$
	$A_2A_2$	White	1	0

(A)  $p$  will decrease, resulting in fixation of  $A_2$   
 (B)  $p$  will decrease, and approach 0 asymptotically  
 (C)  $p$  will reach an equilibrium between 0 and 1  
 (D)  $p$  will increase, approaching 1 asymptotically  
 (E)  $p$  will increase, resulting in fixation of  $A_1$

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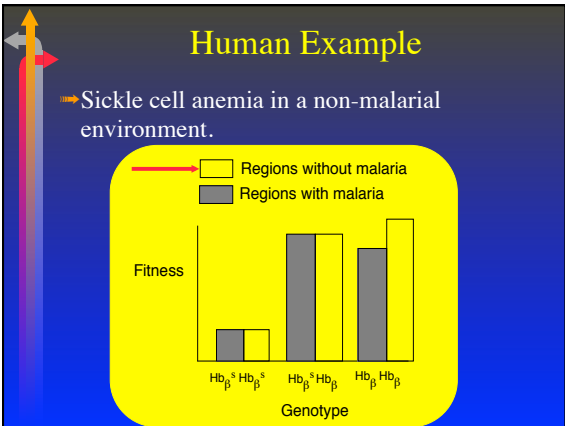
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**Case 5 - Heterozygote superiority**

Case	Genotype	Phenotype	Fitness	Selection coefficient
5	$A_1A_1$	★ Purple	$w_{11}$	$s_{11}$
	$A_1A_2$	★ Pink	1	0
	$A_2A_2$	★ White	$w_{22}$	$s_{22}$

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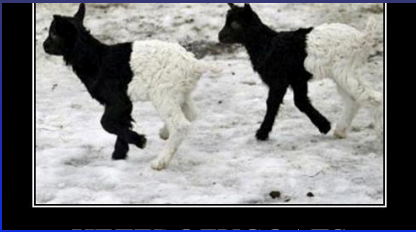
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**Heterozygote superiority...**




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**Case 5 - Heterozygote superiority**  
**What will happen?**

Case	Genotype	Phenotype	Fitness	Selection coefficient
5	$A_1A_1$	★ Purple	$w_{11}$	$s_{11}$
	$A_1A_2$	★ Pink	1	0
	$A_2A_2$	★ White	$w_{22}$	$s_{22}$

(A)  $p$  will decrease, resulting in fixation of  $A_2$   
 (B)  $p$  will decrease, and approach 0 asymptotically  
 (C)  $p$  will reach an equilibrium between 0 and 1  
 (D)  $p$  will increase, approaching 1 asymptotically  
 (E)  $p$  will increase, resulting in fixation of  $A_1$

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### Case 5 - Heterozygote superiority

→ With heterozygote superiority, what is the eventual equilibrium  $p$ ?

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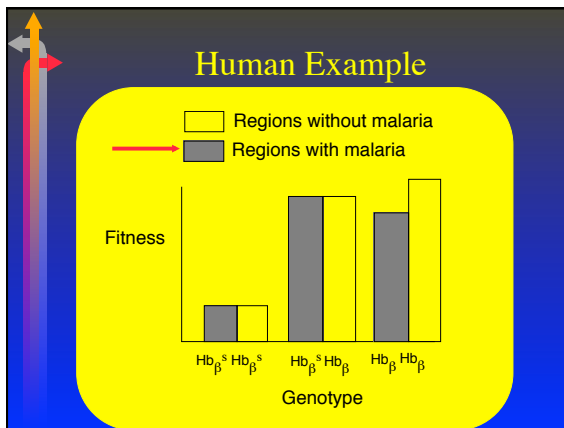
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### Case 6 - Heterozygote inferiority

→  $W_{11}=1+s_{11}$   
→  $W_{12}=1$   
→  $W_{22}=1+s_{22}$   
→  $s$  is a selective 'favor' coefficient (for this case only) because....

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### Case 6 - Heterozygote Inferiority

Case	Genotype	Phenotype	Fitness	Selection coefficient*
4	$A_1A_1$	 Purple	1 or $W_{11}$	$1+s_{11}$
	$A_1A_2$	 Pink	$W_{12} < 1$ (lowest)	1
	$A_2A_2$	 White	1 or $W_{22}$	$1+s_{22}$

Note: s is a selection 'favor' coefficient in this case only...

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### Case 6 - Heterozygote inferiority

Case	Genotype	Phenotype	Fitness	Selection coefficient*
4	$A_1A_1$	 Purple	1 or $W_{11}$	$1+s_{11}$
	$A_1A_2$	 Pink	$W_{12} < 1$ (lowest)	1
	$A_2A_2$	 White	1 or $W_{22}$	$1+s_{22}$

(A) p will decrease, resulting in fixation of  $A_2$   
 (B) p will decrease, and approach 0 asymptotically  
 (C) p will reach an equilibrium between 0 and 1  
 (D) p will increase, approaching 1 asymptotically  
 (E) p will increase, resulting in fixation of  $A_1$

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### Case 6 - Heterozygote inferiority

→ What is the eventual fate of alleles  $A_1$  and  $A_2$ ?

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### Be Able to Answer These Questions!

- When (what case?) is the disfavored allele eliminated?
- When does the disfavored allele persist in low frequencies?
- When is allelic diversity preserved?
- When does initial  $p, q$  matter?

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### Real world selection

- Does evolution follow the pattern predicted by the selection equation in the real world?
- Most famous example: *Biston betularia* - peppered moth, of Britain, studied by H. B. D. Kettlewell.

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### Genetics of Melanism

- One gene, two alleles.  $M$  dominant over  $m$ .  $M$  produces dark pigmentation in dominant homozygote and heterozygote.
- $MM$ =melanic
- $Mm$ =melanic
- $mm$ =typical

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## Demonstration of Selection

→ Clarke and Sheppard (1966) experiment

Phenotype:	Melanic		Typical	
Environment	Exposed	Survived	Exposed	Survived
Dark Background	70	58	70	39
Pale Background	40	24	40	32

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## *Biston betularia*

→ Which case of selection is this?

(A) Selection against the dominant  
 (B) Selection against the recessive  
 (C) Heterozygote superiority  
 (D) Heterozygote inferiority

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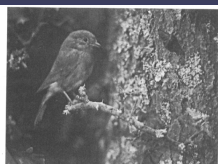

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## *Biston betularia*

→ Which case of selection is this?

(A) Selection against the dominant  
 (B) Selection against the recessive  
 (C) Heterozygote superiority  
 (D) Heterozygote inferiority

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### Contrasting Patterns of Selection

- Selection acts in opposite directions in the two environments.

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### Summary

- In a one-gene, two allele system, there are 6 'cases' of selection
- Most cases of selection result in elimination of disfavored alleles
- Deleterious recessive alleles are difficult to remove *via* selection
- Heterozygote superiority maintains genetic diversity at a locus by favoring individuals that contain both alleles

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