Pulsar statistics: the birthrate and initial spin periods of radio pulsars

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ABSTRACT

We investigate the birthrate and initial spin periods of radio pulsars using the recently revised pulsar distance scale of Cordes et al. Our technique is based on the model-free approach originally suggested by Phinney & Blandford, which makes no assumptions about pulsar luminosity as a function of period, the beaming fraction, or field decay in radio pulsars. It eliminates the large statistical errors inherent in previous methods by restricting the sample to pulsars with luminosities greater than some minimum value. We verify the validity of our approach, and shortcomings of previous methods, by using Monte Carlo simulations on synthetic populations of pulsars which take into account random errors in the pulsar distance scale. We find that small-number statistics and errors in the pulsar distance scale prohibit us from making any firm statements on the birthrate or initial spin periods of pulsars with luminosities less than 10 mJy kpc^2. The birthrate of potentially observable pulsars with luminosities greater than 10 mJy kpc^2 is between 0.6 and 2 per 1000 yr. There is no convincing evidence that any of these pulsars are born spinning slowly. We estimate that there are (1.3 ± 0.2) × 10^4 potentially observable pulsars in the Galaxy with luminosities >10 mJy kpc^2. Application of Biggs's beaming law to our results suggests that a pulsar is born once every 125 to 250 yr in the Galaxy, considerably lower than recent estimates of a supernova rate of 1 per 30 yr, but consistent with earlier estimates of the supernova rate. The local pulsar birthrate is calculated to be between 6 and 12 pulsar kpc^{-2} Myr^{-1}, which is in excellent agreement with Blauuw's estimate of the local production rate of OB stars.

Key words: methods: statistical – pulsars: general – radio continuum: stars.

1 INTRODUCTION

The major problem when attempting to derive the underlying model-free approach originally suggested by Phinney & Blandford (1981) as an alternative to earlier approaches using luminosity models (see for example Gunn & Ostriker 1970; Taylor & Manchester 1977). This new technique involved calculation of the rate at which pulsars flow across the magnetic field–period diagram, that is the pulsar 'current'. The idea was to see if there were sufficient pulsars being born at short periods to sustain the number of pulsars observed at long periods. This was done by using their measured period derivatives and scale factors obtained from \( V/V_{\text{max}} \) computations, where \( V \) is the weighted volume in which a pulsar is potentially observable and \( V_{\text{max}} \) is the total volume of the Galaxy. The purpose of these scale factors is to correct the observed distribution to the distribution of the underlying population.

Applying this analysis to the 'standard model' for pulsar evolution supported by Lyne, Manchester & Taylor (1985) (hereafter LMT), in which all pulsars are born in supernovae and have short initial periods \( P_0 \leq 20 \text{ ms} \), the expected distribution of current as a function of period would initially show a smooth rise until \( P_0 \), then a plateau at periods where

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pulsars are ‘alive’, followed by a decline when pulsars no longer radiate sufficient energy to be detected (see fig. 1 of VN). The birthrate of this population is then simply proportional to the height of the plateau. The major advantage of this approach over previous methods is that it assumes nothing about the emission process or the nature of pulsar spin-down (both of which are poorly understood), and in this sense is model-free. However, it does still require assumptions about the dimensions of the Galaxy, the scaleheight of the pulsar progenitors, pulsar velocities, the interstellar medium, and pulsar surveys in order to compute the scale factors \( (V/V_{\text{max}}) \) necessary to correct the observed distribution to the distribution of the underlying population.

When VN calculated the current they found a discontinuity in the distribution, showing a significant increase in the current at a period of \( \sim 0.5 \) s. They concluded that, in order to account for this anomaly, a large number of pulsars must be ‘injected’ into the population with periods of about 0.5 s, clearly challenging the conventional view that radio pulsars begin their lives with short periods. Initially it was suggested that injection was caused by an incomplete treatment of the selection effects inherent in pulsar surveys. In particular, Manchester, D’Amico & Tuohy (1985) pointed out that any young, short-period pulsars lying close to the galactic plane would suffer extensively from dispersion smearing and pulse scattering, making them difficult to detect with early pulsar searches. However, the failure of the Green Bank phase II survey (Stokes, Taylor & Dewey 1985) to detect any of these pulsars, despite its significantly faster sampling rate, weakened the case against injection. If short-period pulsars have larger beaming fractions than their long-period counterparts, as has been argued (see for example Narayan & Vivekanand 1983; Lyne & Manchester 1988; Biggs 1990), then any case for injection is strengthened.

In view of this debate, Narayan (1987) repeated the pulsar-current analysis with a more careful treatment of selection effects, making use of a larger data base of pulsars than was available to VN. In addition, he attempted to minimize statistical fluctuations in the analysis by assuming that the observed luminosities of pulsars are proportional to the cube root of the rate of loss of rotational energy, i.e. their spin-down luminosity. He concluded that injection was significant for pulsars with high magnetic fields \( (\geq 10^{12} \text{ G}) \), and virtually absent in the sample with low magnetic fields. He suggested that one plausible explanation for this could be if angular momentum is not conserved during the supernova explosion of the progenitor star, but is transferred by magnetic coupling, so progenitor stars with high magnetic fields would produce neutron stars with larger initial spin periods.

Several authors have used Monte Carlo techniques to investigate the problem of the initial spin periods of the pulsar population. The patchy sky coverage of the major surveys means that they cannot be modelled analytically to any satisfactory degree. The treatment of selection effects requires a computer model. In order to model the pulsar population, a large number of pulsars (typically \( \sim 10^5 \)) are introduced into a model galaxy with an initial spatial distribution and allowed to move in the gravitational potential of the galaxy for some finite time. By ‘observing’ the model pulsars using the computer simulations of the original pulsar surveys, a synthetic population can be built up by recording the parameters of all the model pulsars that are ‘detected’. The quality of any model for pulsar evolution can then be assessed by a statistical comparison of the real and model populations ‘observed’.

The results of previous Monte Carlo simulations are inconclusive. Stollman (1987) claims that injection is not required for good fits to the data. Alternatively, Emmering & Chevalier (1989) suggest that the observed data are more consistent with all pulsars being injected with longer periods, but that models without injection cannot be ruled out. More recently, Narayan & Ostriker (1990) have addressed these issues. In their preferred model, there are two populations of pulsars in approximately equal numbers: one population is born spinning rapidly whilst, in the other, pulsars are born with an initial period of between \( \sim 0.5 \) and 1 s. In contrast, Bhattacharya et al. (1992) suggest that models with all pulsars being born with periods of 100 ms are consistent with the data. The main reason these papers give contrasting results lies in their choice of luminosity law. Emmering & Chevalier favoured a law in which young rapidly spinning pulsars are very bright, and therefore required injection in order to account for the lack of such objects in the observed sample. Stollman used a two-stage luminosity law in which the radio luminosity is constant until pulsars reach a given period, after which it declines. The differences between the results of Bhattacharya et al. (1992) and Narayan & Ostriker (1990) are more disturbing because both of these studies used identical modelling of past pulsar surveys. In particular, their opposite conclusions on the necessity of field decay need to be resolved.

When attempting to establish the birth properties of the pulsar population, we are faced with a dilemma. The model-free pulsar-current approach is clearly desirable, as it makes a minimum number of assumptions about some particularly poorly understood aspects of pulsars, but it is subject to large statistical uncertainties. The Monte Carlo techniques are powerful, but do not appear to give ‘unique solutions’ for some issues to which they are applied. In this paper, we use Monte Carlo techniques on synthetic populations of pulsars whose properties we control, to ascertain the validity of the pulsar-current analysis used by VN and Narayan (1987). We then use synthetic populations to develop a powerful refinement of the pulsar-current technique, which more accurately assesses the initial spin periods and birthrate of pulsars. Our method is much less sensitive to statistical fluctuations and distance errors than were previous studies, as we eliminate the few low-luminosity pulsars which dominate the pulsar-current analysis, thus enabling us to use the large number of bright pulsars to make statistically significant statements about the initial spin periods of radio pulsars.

The plan for the rest of this paper is as follows. In Section 2 we describe our assumptions necessary to model the pulsar population, the interstellar medium and the pulsar surveys. In Section 3 the method we use to compute the pulsar-current analysis is described. In Section 4 we assess the shortcomings of previous attempts to resolve the initial spin periods and birthrate of pulsars, and present the results of our new approach, which we verify with synthetic populations of pulsars. Finally, in Section 5 we present our conclusions.
2 MODELLING THE PULSAR POPULATION

One of the main objectives of this paper is to use well-defined synthetic populations of pulsars to test the validity of any statistical method aimed at deriving the initial spin periods and birthrate of pulsars. In order to model the pulsar population, we need to model the distribution of progenitors in the Galaxy, the effects of the interstellar medium, the major pulsar surveys, and how pulsars evolve. The usual approach is to generate a large number of pulsars (typically $\sim 10^5$), allow them to evolve for some finite time, and then ‘observe’ them by using computer models of the major pulsar surveys.

We can estimate the distribution of pulsars in the Galaxy by assuming their initial positions and velocities, and then allowing them to evolve in the galactic potential for a typical pulsar lifetime. The initial state of the population can be represented by a probability distribution function $\Psi$, such that

$$
\Psi(R, z, v_x, v_y, v_z) \, dR \, dz \, dv_x \, dv_y \, dv_z = \Psi_0(R_0) \, dR \, \Psi_0(z) \times dz \, \Psi_0(v_x, v_y, v_z) \, dv_x \, dv_y \, dv_z.
\tag{1}
$$

In this notation, $R$ is the initial galactocentric radius of the pulsar (kpc), $z$ is its initial height above the galactic plane (pc), and the initial velocity has Cartesian components $v_x$, $v_y$ and $v_z$ (km s$^{-1}$). After Narayan (1987), we choose $R_0$ from a Gaussian probability distribution:

$$
\Psi_0(R) \, dR = \frac{1}{\sqrt{2 \pi R_0^2}} \exp\left(-\frac{R^2}{2R_0^2}\right) \, dR.
\tag{2}
$$

Throughout this paper, the galactocentric radius of the Sun is assumed to be 8.5 kpc and the disc scalelength $R_0$ is 4.8 kpc. We choose $z_0$ from a Gaussian distribution with a root mean square height $z_0 = 100$ pc, consistent with the scaleheights of OB stars as the progenitors of the pulsar population. In keeping with previous authors, the initial velocities are selected from a Maxwellian distribution:

$$
\Psi_0(v_x, v_y, v_z) \, dv_x \, dv_y \, dv_z = \frac{1}{(2\pi \sigma_v^2)^{3/2}} \exp\left[-\frac{(v_x^2 + v_y^2 + v_z^2)}{2 \sigma_v^2}\right] \, dv_x \, dv_y \, dv_z,
\tag{3}
$$

with the three-dimensional velocity dispersion $\sigma_v$ fixed at $\sqrt{3} \times 100$ km s$^{-1}$. We have found that our results are insensitive to the exact form of this distribution.

Each pulsar is assigned at age $t$ from a flat distribution with a maximum age $t_{\text{max}}$. In order to reduce the computational time involved in generating the model pulsar population, $t_{\text{max}}$ is set at 25 Myr. The mean birthrate of the $N_{\text{gal}}$ pulsars in the Galaxy is then given by $N_{\text{gal}}/t_{\text{max}}$. We then solve the equation of motion of each pulsar as it moves under an effective gravitational potential $\Phi$ using Poisson’s equation

$$
\nabla^2 \Phi = 4\pi G \rho.
\tag{4}
$$

Here, $\rho$ is the galactic mass distribution function, which we adopt from Kuijken & Gilmore (1989).

Given the position and velocity of each member of the synthetic population, we need a model to account for the deleterious effects of the interstellar medium on pulsar signals. Broad-band pulses travelling through the interstellar medium suffer a dispersive delay caused by the presence of free electrons, which changes the group velocity of the signal. This smears the pulse over a finite receiver bandwidth, since high-frequency signals travel faster and arrive earlier than low-frequency ones. The strength of this effect is directly related to the dispersion measure (DM), which is the integrated electron density along the line of sight to the pulsar:

$$
DM = \int_0^d n_e \, dl \, \text{cm}^{-3},
\tag{5}
$$

where $d$ is the distance to the pulsar (pc) and $n_e$ is the free electron density (cm$^{-3}$). The DMs, together with an electron density model, are the main source of distance estimates for pulsars. In this paper we use the most recently published electron distribution model derived by Cordes et al. (1991),\footnote{After submitting this paper, we became aware of the distance model of Taylor & Cordes (1993). Although this model is more detailed than the Cordes et al. (1991) model, the essential features of both models are similar and the conclusions from this paper remain unchanged.} in which the density of free electrons at any point in the Galaxy is given by

$$
n_e = 0.025 \exp\left(-\frac{|z|}{1 \text{ kpc}}\right) \exp\left(-\frac{R^2}{400 \text{ kpc}^2}\right) + 0.2 \exp\left(-\frac{|z|}{0.15 \text{ kpc}}\right) \times \exp\left(-\frac{(R - 4)^2}{4 \text{ kpc}^2}\right) \, \text{cm}^{-3}.
\tag{6}
$$

Cordes et al. (1991) derived their model by two independent routes: first by calibrating the DMs of pulsars with independently measured distances (from H$\alpha$ absorption techniques, parallax measurements and known supernova remnant associations), and secondly from scattering measurements (see below) of galactic and extragalactic radio sources. Both methods yield the same fit to give the model in equation (6), implying that no large systematic errors are present in the model, although there are random errors estimated to amount to about a factor of 2.

Given the position of each model pulsar in the Galaxy, we first estimate its DM by integrating equation (5) along the line of sight using the electron density model in equation (6). We then account for the random errors by calculating the ‘derived’ dispersion measure $DM_{\text{der}}$ from

$$
\log DM_{\text{der}} = \log DM_{\text{model}} + \Gamma,
\tag{7}
$$

where $\Gamma$ is a suitably normalized Gaussian distribution about the origin with a standard deviation of $\log 2$.

An equally important selection effect for distant pulsars is the scattering of the pulse by electron density variations in the interstellar medium. The difference in path-lengths and therefore in arrival times of the scattered signals means that the observed pulse often has a significant ‘scattering tail’, making detection of the pulsar more difficult. Scattering becomes more severe towards the Galactic Centre, where the density of free electrons increases. The broadening of the pulse due to this effect, the scattering time ($\tau_{\text{scat}}$), is known to be significantly correlated with dispersion measure (see for example Slee, Dulk & Otrupcek 1980). The scattering time
can be estimated either by using a scattering model for the free electrons (see for example Narayan 1987) or from a power-law fit to the DM - \tau_{\text{scint}} correlation (Bhattacharya et al. 1992). We choose the latter approach for convenience, which models this dependence (at 400 MHz) by

\[ \tau_{\text{scint}}(\text{ms}) = 10^{-4.62} + 1.14 \log(\text{DM}) + 10^{-9.22} + 4.46 \log(\text{DM}). \] (8)

To calculate the scattering time at higher observing frequencies, we use the simple \( \nu^{-4.4} \) scaling law, after Romani, Narayan & Blandford (1986). We account for the spread about this relation in a similar fashion to the DM model.

The initial periods and magnetic fields of the pulsars are selected from Gaussian distributions about mean values of \( \log P_0 \) and \( \log B_0 \), with standard deviations (in the logarithm) of \( \sigma_{P_0} \) and \( \sigma_B \) respectively. In this paper we consider two models, which we call A and B. Model A is based on the Stollman (1987) model, in which pulsars have short initial periods such that \( \log P_0 = -1.65 \). In model B, pulsars are born spinning slower on average, so that \( \log P_0 = -0.4 \) with a standard deviation in the log of 0.35, where \( P_0 \) is measured in seconds. This is similar to the preferred model of Emmering & Chevalier (1989). Good fits to the observed data in both models are obtained with \( \log B_0 \) (in gauss) set at 12.46 and 12.30 with a standard deviation, again in the log, set at 0.31 and 0.35 for models A and B respectively.

The decay time-scale for the magnetic field is a contentious issue in pulsar astronomy. Baym, Pethick & Pines (1969), proposed, on theoretical grounds, that the magnetic field is retained indefinitely within the neutron star. However, an early statistical study by Gunn & Ostriker (1970) concluded that magnetic fields decayed on short (Myr) timescales. Observational evidence from proper motion studies suggested that pulsars' magnetic fields decayed on a timescale of \( \lesssim 10 \text{ Myr} \) (Lyne, Anderson & Salter 1982), and in their study Narayan & Ostriker (1990) have concluded that populations without field decay can be ruled out with a high confidence level. However, Bhattacharya et al. (1992) have demonstrated, using a Monte Carlo approach, that the observed properties of the pulsar population are consistent with much longer decay times (\( \gtrsim 100 \text{ Myr} \)). In this paper, the issue of field decay is unimportant. The model-free pulsar current analysis makes no assumptions about field decay, and should be equally applicable to populations with long or short field decay times. For convenience, for our synthetic population of pulsars we assume that the magnetic fields of pulsars decay from their initial values exponentially, i.e.

\[ B(t) = B_0 \exp \left( -t/t_\text{d} \right), \]

where \( t_\text{d} \) is the decay time-scale which we set at 10 Myr (although our results are relatively unaffected by this assumption).

For the period evolution of our pulsars, we assume that the energy loss process is dominated by magnetic dipole radiation. In this case, Ostriker & Gunn (1969) have shown that

\[ P \dot{P} = \frac{8\pi^2 r_0^6}{3c^3} B^2 \sin^2 x. \] (9)

where \( I \) and \( r \) are the moment of inertia and radius of the star respectively, \( x \) is the angle between the spin and magnetic axes, and \( c \) is the speed of light. We assume that \( \sin x = 1 \) (consistent with the observational evidence of Lyne & Manchester 1988), and adopt the canonical values of the parameters, \( I = 10^{45} \text{ g cm}^2 \) and \( r = 10^6 \text{ cm} \), in our models.

Integration of equation (9) assuming exponential field decay yields the well-known period-evolution expression:

\[ P^2 + \dot{P}^2 + 8\pi^2 r_0^6 \frac{1}{3c^3} t_\text{d} B_0^2 \left( 1 - \exp(-2t/t_\text{d}) \right). \] (10)

Given the period and period derivative, we use a power-law expression to determine the luminosity. The power law was first suggested by Lyne, Ritchings & Smith (1975), and gives the luminosity \( L \) in units of mJy kpc\(^2\) by

\[ L = \log \gamma + \alpha \log P + \beta \log \dot{P}, \] (11)

where the period \( P \) is expressed in s, \( \dot{P} \) is in \( 10^{-15} \text{ s s}^{-1} \) and \( \gamma, \alpha \) and \( \beta \) are constants. These units are assumed throughout the rest of this paper. In model A, we adopt the two-stage luminosity law used by Stollman (1987),

\[ L_A = \begin{cases} 10 & \text{for } P^{-1.5} \dot{P}^{0.5} > 10 \text{ s}^{-1.5}, \\ P^{-1.5} \dot{P}^{0.5} & \text{for } P^{-1.5} \dot{P}^{0.5} \leq 10 \text{ s}^{-1.5}, \end{cases} \] (12)

while in model B, pulsars obey the luminosity law used by Emmering & Chevalier (1989): \( L_B = \gamma_b P^{-1.61} \dot{P}^{0.64} \).

The constants \( \gamma_A \) and \( \gamma_B \) are adjusted until a satisfactory fit is obtained to the observed pulsar luminosity distribution. In our final simulations, we set \( \gamma_A = 2.8 \) and 3.5 mJy kpc\(^2\) respectively.

To account for the large scatter in the observed luminosities of pulsars with similar periods and magnetic field strengths, we spread the model luminosity as a Gaussian distribution with a standard deviation in the logarithm, in an analogous manner to the DMs and scattering times. We find that, with a standard deviation in the log of 0.8, the spread in luminosities is modelled satisfactorily.

Given the luminosity of a pulsar and its position in the model galaxy, the flux density at 400 MHz is given by \( S_\text{dip} = (L/d^2) \text{ mJy} \), where \( d \) is the geometrical distance to the pulsar in kpc. Since the distances to the real pulsars are limited in accuracy by fluctuations in the free electron distribution in the Galaxy, given the model DM from equation (7) we calculate the 'derived distance' \( d_\text{der} \) of a model pulsar using equations (5) and (6). This allows us to simulate distance errors which affect the luminosities. The 'derived luminosity' is therefore \( L_\text{der} = S_\text{dip} d_\text{der}^{-2} \).

Finally, we simulate the nine major pulsar surveys on these model populations to build up model observed samples. The surveys considered are the Jodrell Bank 'A' survey (Davies, Lyne & Seiradakis 1972, 1973), the U-Mass Arcibo survey (Hulse & Taylor 1974, 1975), the second Molonglo survey (Manchester et al. 1978), the U-Mass NRAO survey (Damashek, Taylor & Hulse 1978), the Princeton NRAO survey phase I (Dewey et al. 1985) and phase II (Stokes, Taylor & Dewey 1985), the Jodrell Bank 'B' survey (Clifton et al. 1992), the Princeton Arcibo survey (Fruchter 1989), and the Parkes survey (Johnston et al. 1992). These surveys detected a total of 463 pulsars, representing over 80 per cent of the known population.

To be counted as detectable, a pulsar must be beaming towards the Earth, must lie within the search area, and must have a flux density above the minimum detectable by the
observing system. Various beaming models have been proposed which affect the detectable fraction of pulsars as a function of period (see Section 3). We take these models into account when making conclusions about the real pulsar population in Section 4, but in our model populations, for simplicity, we assume that pulsars beam to the entire sky. For most surveys, the search area is well defined by galactic latitude and longitude, but the Princeton NRAO searches used random positions selected from a larger grid. To be more consistent, we use the actual beam positions from these two surveys.

Pulsars lying within the search area are assigned a random distance $r$ from the nearest beam centre. For a telescope beam with a Gaussian power pattern, the flux degradation factor $(\delta_{\text{beam}})$ suffered by a pulsar lying a distance $r$ off the beam centre is given by

$$\delta_{\text{beam}} = \exp(-r^2/f_{\text{beam}}), \quad (14)$$

where $f_{\text{beam}} = b/(2\ln 2)$ for a telescope beamwidth with a FWHM of $b$ degrees. For some surveys (for example Johnston et al. 1992), the telescope beam drifted across the sky rather than tracking a given area. In this case, the flux degradation suffered by a pulsar covered by a scan starting at a position $x_0$ with a drift rate $\dot{x}$ during an integration time $t_{\text{int}}$ is given by

$$\delta_{\text{beam}} = \exp(-y^2/f_{\text{beam}}) \int_0^{t_{\text{int}}} \exp\left[-(x_0 + \dot{x}t)^2/f_{\text{beam}}^2\right] dt, \quad (15)$$

where $y$ is the declination offset between the scan and the pulsar in degrees. The effective flux density $(S_{\text{eff}})$ of the pulsar at the telescope is therefore given by

$$S_{\text{eff}} = S_{\text{tot}} \delta_{\text{beam}} \left(\frac{\nu}{400 \text{ MHz}}\right)^\zeta. \quad (16)$$

The observed spectral index $\zeta$ of pulsars is typically $\sim -1.5$. To model this effect, we assign each pulsar a spectral index from a Gaussian distribution centred on $-2.1$ with a standard deviation $\sigma = 0.5$.

After Dewey et al. (1984), the theoretical minimum flux density detectable by a telescope is

$$S_{\text{min}} = C(T_{\text{rec}} + T_{\text{sky}}) \sqrt{\frac{W_r}{G N_p A\nu t_{\text{int}}}} \sqrt{\frac{W_r}{P - W_r}}. \quad (17)$$

In this expression, $C$ is a constant $\sim 10$, guaranteeing a detection threshold above the $7a$ level, $T_{\text{rec}}$ and $T_{\text{sky}}$ are the system and sky noise temperatures (K) respectively, $G$ is the antenna gain (K Jy$^{-1}$), $N_p$ is the number of polarizations used, $A\nu$ is the receiver bandwidth (MHz) and $t_{\text{int}}$ is the integration time (s), $W_r$ is the observed pulse width, which we discuss below.

The detection of pulsars lying towards the Galactic Centre is hampered by the high sky background temperature ($T_{\text{sky}} \sim 300$ K) in that region, increasing the effective system noise temperature. We obtain the sky background temperature at 408 MHz from the all-sky catalogue of Haslam et al. (1982). Using data from surveys at 1420 MHz, Lawson et al. (1987) have concluded that the spectral index of the background temperature is steep, with a value of $\sim -2.8$. We adopt this value to calculate the appropriate sky background temperature for the high-frequency pulsar surveys of the galactic plane (Johnston et al. 1992; Clifton et al. 1992).

The duty cycle of the intrinsic pulse is assumed to be 4 per cent, corresponding to a width $W=0.04P$. To model the effects of pulse broadening due to instrumental effects, dispersion and scattering, we use the method described by Dewey et al. (1984), representing the broadening pulse width $(W_r)$ by the following expression:

$$W_r^2 = W^2 + \left(\tau_{\text{amp}} \frac{DM}{DM_0}\right)^2 + \left(\frac{t_{\text{amp}} |DM - DM_0|}{DM}\right)^2 + \tau_{\text{scatt}}^2. \quad (18)$$

The second term represents the broadening due to sampling effects; here $\tau_{\text{amp}}$ is the effective sampling time which takes into account details of the system hardware such as anti-aliasing filters. The third term arises from the dispersion broadening across individual receiver channels, where $DM_0$ is the dispersion measure at which the smearing of the pulse in one channel is equal to the data sampling interval $(t_{\text{amp}})$. The fourth term is due to dedispersion during the search being carried out at $DM_0$, a value which may differ from the actual $DM$ of the pulsar. The final contribution to the observed pulse is from the interstellar scattering time $(\tau_{\text{scatt}})$ discussed earlier. Pulsars with an excessively broad pulse $(W_r \geq P)$ cannot be resolved against the background noise and are therefore undetectable. Using equations (16)-(18), we determine whether a model pulsar’s flux is above the detection threshold. The parameters of these ‘detected’ pulsars are recorded in order to build up a synthetically observed population.

3 THE PULSAR-CURRENT ANALYSIS

In this section, we briefly outline the theory of pulsar current. A more detailed description can be found in Pinney & Blandford (1981), VN and Narayan (1987).

The narrow beam of emission from pulsars means that they are not visible from all points in the Galaxy. To model this effect, we introduce the beaming fraction $(f)$ of sky covered by the pulsar beam in one rotation. Assuming a random distribution of magnetic inclination angles, Emmering & Chevalier (1989) have shown that

$$f = (1 - \cos \theta) + \left(\frac{\pi}{2} - \theta\right) \sin \theta, \quad (19)$$

where $\theta$ is the half-angle of the emission cone. Typically, $\theta \sim 10^\circ$, corresponding to a beaming fraction of $\sim 20$ per cent (Taylor & Manchester 1977).

It is generally believed that $\theta$ (and therefore $f$) is period-dependent, since shorter period pulsars appear to have wider beams and are therefore potentially visible from a larger portion of the Galaxy than are longer period pulsars. This of course strengthens the argument for injection. However, there is much debate as to the exact shape and evolution of the beam. Originally, Narayan & Vivekanand (1983) suggested meridional (north–south) elongation, whereas
Lyne & Manchester (1988) claimed the emission cone to be essentially circular. More recently, Biggs (1990) has reassessed this question using the Lyne & Manchester database, and has concluded that meridional compression is more likely. It should be noted that all pulsar beaming models are restricted by assumptions made about pulse geometry.

When deriving birthrates for the real pulsar population, we adopt the most recent beaming model (Biggs 1990), which gives the opening half-angle of the emission region as

$$\theta = \frac{6.2}{\sqrt{P}} \text{deg},$$

where $P$ is measured in s. We avoid where possible the above debate on what is the correct beaming law, by quoting the "potentially observable" birthrates (i.e. $f = 1$) which are free from any assumptions about beaming or pulse geometry.

We define the current ($J_P$) of pulsars assumed to be in steady state and having gradually increasing periods as a function of period by the following expression:

$$J_P(P) = \int_{L} N_{gal}(P, \hat{P}, L) \hat{P} d\hat{P} dL,$$

where $N_{gal}(P, \hat{P}, L)$ represents the true distribution of pulsars in the Galaxy as a function of $P$, $\hat{P}$ and $L$. We relate this function to the observationally selected distribution, $N_{obs}(P, \hat{P}, L)$, by

$$N_{gal}(P, \hat{P}, L) = N_{obs}(P, \hat{P}, L) \frac{\xi(P, L)}{f(P)}.$$

This allows for a period-dependent beaming fraction $f(P)$ discussed above and a scaling factor $\xi(P, L)$, which is the ratio of the volume (weighted by pulsar density) of the Galaxy to the volume (also weighted by pulsar density) in which the pulsar would be detected. Thus low-luminosity pulsars which are visible out to only a few kiloparsecs from the Sun with the present surveys will have much larger scale factors compared to brighter pulsars. To compute the scale factor of each pulsar, we place it at each of the $10^5$ positions we generate in the model Galaxy to simulate the distribution of pulsars, recalculate its apparent flux given its luminosity and position, and attempt to reobserve it using the same surveys as described above, recording the number of detections. For a galaxy with $10^5$ positions, the scale factor of a pulsar detected $N_{det}$ times is then $10^5/N_{det}$.

To improve the statistics, we calculate the mean current ($\bar{J}_P$) for $n$ pulsars within a given period range $\Delta P$; after Narayan (1987), this is given by

$$\bar{J}_P = \frac{1}{\Delta P} \sum_{i=1}^{n} \frac{\hat{P}_i \xi_i}{f_i}.$$

The number of potentially observable pulsars in the Galaxy with luminosities above the minimum luminosity of our sample is simply the sum of all the scale factors. An estimate of the statistical error only is then given by the square root of the sum of the squares of the scale factors (VN).

The real observed pulsar sample we use in our analysis consisted of all pulsars detected by the above nine surveys. In some cases, a distance of less than 30 kpc is unobtainable using the Cordes et al. (1991) electron density model, due to the finite scaleheight of free electrons assumed. Removal of these pulsars from the sample leaves us with 421 pulsars, of which 368 have published values of $\hat{P}$. Unpublished values of $\hat{P}$ for 44 of the newly discovered pulsars from the Johnston et al. (1992) survey have been kindly supplied to us by S. Johnston (private communication), which leaves us with 412 pulsars in our sample. To our knowledge, this is the largest observed sample of pulsars used in a study of this kind. The pulsars we have omitted from the sample have large luminosities, and hence very small scale factors, and therefore do not significantly affect our results.

### 4 RESULTS

We begin this section by computing the pulsar current on our sample of 412 pulsars. Finding the results unsatisfactory, we then use the Monte Carlo models of pulsar populations described in Section 2 to investigate the usefulness of the method in determining the initial spin periods of radio pulsars. We then investigate whether the use of luminosity models to increase the statistical significance of the pulsar-current analysis is valid. Finding that all luminosity models are extremely uncertain, we then suggest a new technique to improve the 'model-free' pulsar-current analysis. Finally, we use this new technique to investigate the birth properties of the observed pulsar population.

#### 4.1 The model-free pulsar current analysis

We applied the model-free pulsar-current analysis to our observed pulsar sample to determine whether conclusions reached by VN and Narayan (1987), using a smaller sample of pulsars and a different distance model, are still valid. The distribution of pulsar current we obtain from this method is shown in the top panel of Fig. 1, and has three distinct humps of similar magnitude. A naive interpretation of these humps, neglecting the large error bars, would conclude an apparent injection of pulsars at periods of $0.3$, $1.0$ and $2.3$ s. This conclusion cannot be taken seriously, since closer scrutiny of the data shows that each hump is produced by a single pulsar and is simply an artefact of small-number statistics. The 0.3-s hump corresponds to PSR B0656+14, at 1 s the distribution is dominated by PSR B1916+14 and at 2.3 s it is dominated by PSR B0154+61. Based on the Cordes et al. (1991) distance model, these three pulsars are all nearby and have luminosities well below 10 mJy kpc$^2$, which give rise to their large scale factors. This suggests that, as a probe of the initial spin periods of radio pulsars, the model-free current analysis is very poor, and dominated by a handful of low-luminosity pulsars.

Given that the basic pulsar current analysis is dominated by small-number statistics, in this section we use the Monte Carlo simulations described in Section 2 to assess whether a similar effect can be expected in a synthetic population where the parameters of the model are well understood. The model parameters we used are summarized in Tables 1 and 2. Although the luminosity law and initial period distribution are significantly different in both models, they produce plausible fits to the $P$, $B$, $L$, $d$ and $DM$ distributions of the observed pulsar population.
The distributions of pulsar current derived using the model-free approach are shown in the middle and lower panels of Fig. 1 for models A and B respectively. Model A is intended to represent a population which has all pulsars born spinning rapidly, whereas model B represents one in which they are injected. In both cases, the derived pulsar current is a poor indicator of both the initial spin periods of the pulsars and the true birthrate. These distributions show similar features to those observed in the real pulsar-current distribution, being dominated by a few nearby low-luminosity pulsars.

We can understand what causes the humps in terms of the Cordes et al. (1991) distance model. For nearby pulsars, \( d \ll DM \) and the DMs of our model pulsars are scattered about the true model value typically by a factor of \( \sim 2 \). For half of these pulsars, then, the apparent luminosity is four times lower than the true luminosity, which increases their scale factors by a factor of \( \sim 8 \). The relative numbers of high-luminosity and low-luminosity pulsars in the observed sample are such that, once their scale factors are computed, these dominate the sample.

Given that the true birthrate of both model populations is 0.4 pulsars century\(^{-1}\), we can compare these with estimates from the plateaus of the derived pulsar-current distributions. The inferred birthrates from Fig. 1 are 4 and 2 pulsar century\(^{-1}\) for models A and B respectively, which are clearly large overestimates, mainly because of the incorrect luminosities inferred from their uncertain distances.

The dominance of the current by the low-luminosity pulsars is best shown in Fig. 2, where we plot the integrated pulsar current against derived luminosity for the real and model populations. At luminosities of \( \sim 10 \) mJy kpc\(^2\), more than 80 per cent of the total current contribution has been reached in both the model and real cases. Below this luminosity, the current contribution is highly erratic. In summary, the basic pulsar-current analysis cannot be used to infer the birthrate or initial spin periods of the pulsar population without significant improvement.

### Table 1. Summary of the parameters defining the 'Galaxy' of pulsars searched by the survey simulations. The parameter definitions are given in the text.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{gal} )</td>
<td>100,000</td>
</tr>
<tr>
<td>( R_{0} ) (kpc)</td>
<td>4.8</td>
</tr>
<tr>
<td>( r_{0} ) (pc)</td>
<td>100.0</td>
</tr>
<tr>
<td>( \sigma_{v} ) (km s(^{-1}))</td>
<td>100.0</td>
</tr>
<tr>
<td>( t_{max} ) (Myr)</td>
<td>25.0</td>
</tr>
</tbody>
</table>

### Table 2. The parameters defining the initial period and magnetic field distributions, the field decay time-scale, the spectral index distribution and the luminosity in models A and B.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model A</th>
<th>Model B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \log P_{0} ) (s)</td>
<td>−1.65</td>
<td>−0.40</td>
</tr>
<tr>
<td>( \sigma_{\log P} )</td>
<td></td>
<td>0.35</td>
</tr>
<tr>
<td>( \log B_{0} ) (G)</td>
<td>12.46</td>
<td>12.30</td>
</tr>
<tr>
<td>( \sigma_{\log B} )</td>
<td>0.31</td>
<td>0.35</td>
</tr>
<tr>
<td>( t_{0} ) (Myr)</td>
<td>10.00</td>
<td>10.00</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>2.10</td>
<td>2.10</td>
</tr>
<tr>
<td>( \sigma_{\zeta} )</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>2.85</td>
<td>3.50</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>−1.50</td>
<td>−1.61</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.50</td>
<td>0.64</td>
</tr>
<tr>
<td>( \sigma_{\log L} )</td>
<td>0.80</td>
<td>0.80</td>
</tr>
</tbody>
</table>

### 4.2 Use of a luminosity model

Narayan (1987) attempted to improve the current analysis by replacing the apparent luminosities of the pulsars with model luminosities based upon their periods and period derivatives, and recomputing their scale factors. At this point, the current analysis ceases to be model-free, and all subsequent conclusions depend upon the validity of the assumed luminosity.

\(^{2}\)Note that, even though the true birthrate in model B is 0.4 pulsar century\(^{-1}\), the true pulsar-current distribution for model B in Fig. 1 gives only a lower bound of \( \sim 0.2 \) pulsar century\(^{-1}\) since, in this model, the period ranges for pulsar births and deaths overlap, producing a net decrease in the observed pulsar current (see VN).
law. Narayan assumed that the luminosity of pulsars was given by a power law of the form in equation (11), i.e. $L \propto P^{-1} P^{1/3}$. This law implies that pulsars' radio luminosity is proportional to the cube root of their spin-down luminosity, which has no physical basis but is often used in population studies (see for example Narayan 1987; Bhattacharya et al. 1992). It is similar to the power-law fits to the observed luminosities by VN and Prószynski & Przbycień (1984).

The luminosities derived from the Narayan (1987) luminosity law are plotted against the observed luminosities for the known pulsar sample in Fig. 3. There is clearly an enormous scatter about the suggested relation; much weaker power-law dependences upon the spin-down luminosity are not excluded, and are perhaps even suggested by the data. However, note that Emmering & Chevalier (1989) have demonstrated that the underlying luminosity law is not necessarily that obtained by a fit to the observed luminosities. A law which overestimates the luminosity of the short-period, large-$P$ pulsars induces a systematic bias into the luminosities and scale factors of these pulsars, and hence into the derived information about birthrates and injection. For example, PSR B1154−62 has a derived luminosity which is two orders of magnitude higher than the value predicted from the power law, which reduces its scale factor by a similar factor. On the other hand, PSR B0114+58 has a derived luminosity almost two orders of magnitude below the power-law prediction. In view of the uncertainties which surround any luminosity law, any conclusions based upon it are extremely model-dependent.

Gunn & Ostriker (1970) proposed a law of the form $L \propto P^{4}$, which was subsequently adopted by LMT as it provided the best fit to their analytically derived distributions. We plot the luminosities derived from this law against the distance-derived luminosities in Fig. 3. The large scatter of power-law luminosities for pulsars with similar derived luminosities also makes the use of this law completely inappropriate to describe the observed luminosities of pulsars. For example, PSR B0818−41 has its luminosity underestimated by over three orders of magnitude, and the luminosity of PSR B0154+61 is overestimated by the same factor. Although this law did not affect LMT's birthrate estimates, it was central to their estimates of the initial magnetic field strengths and the field decay time-scale. We therefore suggest that their conclusions about these matters are unreliable.
We have performed a fit to the present data set of 412 pulsars with luminosities derived using the Cordes et al. (1991) distance model, which yields \( \gamma = 142.81 \pm 16.22 \) mJy kpc\(^2\), \( \alpha = -0.68 \pm 0.12 \) and \( \beta = 0.28 \pm 0.05 \). This is very different from previous fits by VN and Prózyński & Przybyszewski (1984), largely because of the distance model used in this paper, and the inclusion of more pulsars in the observed sample. Our results suggest that the dependence of the luminosity upon period and period derivatives is weaker than has previously been estimated.

We conclude by noting that the large scatter of luminosities about the simple \( P - \dot{P} \) power laws suggests that other factors affect the observed luminosity (for example, geometric effects). At the present time, there appears to be no satisfactory model to account for the observed luminosities of pulsars.

### 4.3 An improvement to the model-free approach

For the remainder of this paper, we assume nothing about luminosity laws, and develop a method to investigate the initial spin periods and birthrate of pulsars which uses the observed luminosities of pulsars. To reduce the problems with small-number statistics in the analysis, we exclude all pulsars with derived luminosities below \( L_0 \). On the basis of Fig. 2, a suitable choice appears to be \( L_0 = 10 \) mJy kpc\(^2\). Above this value, the integrated pulsar-current curve is much smoother and free from the small-number statistics which plagued our earlier attempts to derive the pulsar current. The reason for this is just that the observed sample is dominated by bright pulsars. Our birthrate estimates therefore apply only to pulsar populations born with luminosities above \( L_0 \), and we define the death of a pulsar as being when its luminosity falls below \( L_0 \). Fig. 4 shows pulsar-current histograms for the two model populations after making this selection, along with the true pulsar current which we can compute (see Section 3). This has greatly improved matters, since the form of the pulsar-current distribution is now clearly correlated with the initial spin periods of each population, despite our deliberate introduction of distance errors into the model pulsars. In both models, birthrates from the plateaus of the current distributions are in excellent agreement with the plateaus of the true pulsar-current distributions, and the initial spin periods so derived reflect their true initial values. Thus, at least for our model pulsars, the true birth properties of a model pulsar population can be predicted with reasonable accuracy using this approach. Encouraged by this success with the model populations, we now apply this approach to investigate the birth properties and luminosity function of the real pulsar population above \( L_0 \).

Figs 5 and 6 show the distributions of pulsar current we have obtained after excluding pulsars from our sample with luminosities below \( L_0 = 10 \) mJy kpc\(^2\) and \( 10 \) mJy kpc\(^2\). For the potentially observable population in Fig. 5, the distributions imply that all pulsars born with luminosities above \( L_0 \) have short initial spin periods (\( \geq 100 \) ms). The distributions derived assuming the period-dependent beaming model of Biggs (1990) are shown in Fig. 6. We have also included for comparison the distributions derived assuming the Lyne & Manchester (1988) model (dashed line) and the Narayan & Vivekanand (1983) model (dotted line). These distributions all show the same shape, and for convenience we will deal with the Biggs (1990) model throughout the rest of this paper. Fig. 6 shows a slight increase in the pulsar current at periods near \( \sim 500 \) ms, but it is not statistically significant. There is certainly no evidence that injection is occurring for pulsars born with luminosities above \( 10 \) mJy kpc\(^2\). From the plateaus of these distributions, we estimate the birthrate for pulsars born with luminosities above \( L_0 \) to be \( 0.13 \pm 0.07 \) pulsar century\(^{-1}\) for those which are potentially observable, and \( 0.60 \pm 0.20 \) century\(^{-1}\) for the total population if we use the Biggs (1990) beaming model.

Even when scaling with a period-dependent beaming fraction, we find no convincing evidence for the existence of a second population of injected pulsars born with luminosities above \( L_0 \) as proposed by VN and Narayan (1987). Our results are certainly not inconsistent with the results of Narayan & Ostriker (1990), in which they suggest that half of all pulsars are injected, although at luminosities \( > 100 \) mJy kpc\(^2\) there is no evidence for injection and this is where the statistics of our method are at their best. Proponents of injection may argue that we have simply excluded all the
'injected' pulsars, as these are all low-luminosity objects. However, as we have tried to show, the only evidence for these injected pulsars comes from questionable statistics in the first place.

We are confident from the tests on the model pulsar populations that our luminosity exclusion technique reflects the birth properties of pulsars above $L_0 = 10$ mJy kpc$^2$. On this basis, model A (Stollman 1987) is the more realistic of the two models we have considered. For pulsars born with luminosities below $L_0$, the effects of small-number statistics and distance uncertainties prevent us from drawing any conclusions about the initial spin periods or even the existence of such pulsars.

It is worth noting that the only evidence for injection appears after applying beaming corrections. The power-law dependence of the beaming fraction upon period is very uncertain, and there is some danger that application of a beaming law is completely inappropriate. It may be the case that the flux received from a pulsar is a smooth function of the viewing angle. Pulsars observed a long way from the centre of the beam may have substantially smaller fluxes than those observed near the centre, but any so detected would have much larger scale factors. If this were the case, there would be no need to apply a 'beaming law', as their larger scale factor would correct for their luminosity dependence upon viewing angle. Thus application of a beaming law may over-compensate for beaming effects.

In a recent paper, Bailes & Kniffen (1992) demonstrated that, in order for radio pulsars to contribute significantly to the galactic gamma-ray emission, their initial spin periods must be small, i.e. about 20 ms. Our results therefore do not exclude the possibility that much of the galactic gamma-ray flux could originate from pulsars.

### 4.4 The luminosity function and birthrate of radio pulsars

Fig. 7 shows the luminosity function we obtain, given the scale factors of pulsars with derived luminosities above $L_0 = 10$ mJy kpc$^2$ for potentially observable pulsars (upper panel), and assuming the Biggs (1990) beaming model (lower panel). Comparison of these distributions with the observed...
The derived luminosity distributions for the real pulsar population (solid lines) for potentially observable pulsars (upper panel) and after beaming corrections (lower panel). The observed luminosity distributions are also plotted (dashed lines), highlighting the observational bias towards the less common bright pulsars in pulsar surveys.

Figure 7. Derived birthrates as a function of luminosity for model A. The upper panel was calculated assuming a perfect distance model (no uncertainties), while in the lower panel distance uncertainties are accounted for. The solid line marks the birthrate of the population; the dashed line represents the true birthrate above the given luminosity. The effect of distance errors is to induce a systematic increase in the derived birthrate at luminosities below $\sim 10$ mJy kpc$^2$, by a factor of $\sim 2.5$.

The total galactic population of pulsars with luminosities greater than $L_0 = 10$ mJy kpc$^2$, estimated using the Biggs (1990) beaming model, is $(7.34 \pm 1.06) \times 10^4$. The slope $d \log N / d \log L$ of both distributions is approximately $-1$. Again, because of distance uncertainties and small-number statistics, the shape of the luminosity function below $L_0$ is highly uncertain.

So far, we have only been able to estimate the birthrate of pulsars born with luminosities above $L_0$, since the statistics of pulsars below this level are significantly biased by uncertainties in the distance scale. To demonstrate this, in Fig. 8 we have plotted the derived birthrate from the pulsar-current analysis as a function of $L_0$ between 1 and $10^4$ mJy kpc$^2$ for the pulsar population in model A, together with the true birthrate as a function of initial luminosity (dashed line). In these diagrams, the plotted birthrate represents the total birthrate of pulsars with luminosities greater than $L_0$. The upper panel shows the hypothetical case in which the distances to pulsars are known precisely. In this situation, the agreement between the derived birthrates and the curve is excellent. The lower panel shows the same population after inclusion of our modelling of distance uncertainties. From these plots, we see that the net effect of these uncertainties is to overestimate systematically the birthrate at the low-luminosity end by a factor of $\sim 2.5$, whereas derived birthrates at higher luminosities are essentially correct. This is for two reasons. The distribution of the high-luminosity pulsars is more disc-like than that of the low-luminosity ones, because they are generally younger and confined to the galactic disc.
are only proportional to the errors in their distances squared, instead of cubed as is the case for the lower luminosity pulsars. Furthermore, beyond a certain luminosity, pulsars are visible almost anywhere in the Galaxy, making their distances irrelevant to the birthrate estimates.

In Fig. 9, the birthrate \( BR \) as a function of \( L_0 \) is plotted for potentially observable pulsars (top panel), and using the Biggs (1990) beaming law (lower panel). We find that the data above 10 mJy kpc\(^2\) are well described by the expression

\[
\log BR(L > L_0) = \log BR_0 - \frac{(\log L_0)^2}{\eta},
\]

where \( \eta = 2.5 \) and the asymptotic birthrate \( BR_0 \) is 0.22 and 0.79 births per century for potentially observable pulsars and assuming the Biggs (1990) beaming law respectively. We now make a very important observation from Fig. 9. In order to explain the properties of the observed pulsar population, there is no need for any pulsars to be born with luminosities below \( \sim 30 \) mJy kpc\(^2\), although the existence of a significant population of low-luminosity pulsars cannot be ruled out.

In the past, many attempts have been made to determine the birthrate of the pulsar population. Estimates have ranged between one pulsar born every 6 yr (Taylor & Manchester 1977) down to one birth every 130 yr (LMT), with many estimates between (see for example Gunn & Ostriker 1970; VN; Narayan 1987; Narayan & Ostriker 1990). The large range in these estimates is due to uncertainties in the pulsar distance scale, the beaming fraction and the method used. Our birthrate estimate is only a lower limit, because there is insufficient evidence to comment on pulsars born with very low luminosities, but it is significantly lower than most of the above values. This is mainly due to our adoption of the Cordes et al. (1991) distance model, which has increased the distance to most pulsars, so increasing their derived luminosities and reducing their numbers.

The pulsar birthrate derived in this paper is consistent with the estimate of Clark & Caswell (1976) for the formation rate of radio supernova remnants of once per 150 yr if half of all supernovae produce pulsars. This is much lower than the rate derived from historical supernovae in the Galaxy by Clark & Stephenson (1977) of one event per 30 yr which, at the time, was consistent with the pulsar birthrate (Taylor & Manchester 1977). The discrepancy with the Clark & Caswell (1976) estimate forced Clark & Stephenson (1977) to suggest that only 1 in 5 galactic supernovae leave long-lived remnants. In the light of our new birthrate estimates, there is no need for this somewhat ad hoc assumption.

Since the pulsars used in our birthrate calculations reside mainly in the vicinity of the Sun, we can make a robust calculation of the birthrate of pulsars per unit area in the local solar neighbourhood. Given the radial distribution we used to model the Galaxy in equation (2), the birthrate of potentially observable pulsars in the local solar neighbourhood is between 1 and 3 pulsar kpc\(^{-2}\) Myr\(^{-1}\). Assuming the Biggs (1990) beaming model, we calculate the total birthrate of pulsars in the solar neighbourhood to be between 6 and 12 pulsar kpc\(^{-2}\) Myr\(^{-1}\). This is in excellent agreement with the estimate of the local production rate of OB stars by Blaauw (1987) of between 5 and 16 OB stars kpc\(^{-2}\) Myr\(^{-1}\). The uncertainty in Blaauw’s estimate depends on the lower limiting stellar mass required to form a neutron star, which ranges between 6 and 8 M\(_{\odot}\). In addition, since this rate deals with the local population of OB stars, it is less prone to the surface brightness selection effects which plague estimates of the galactic supernova rate.

As we have stressed repeatedly in this paper, our birthrate estimates apply only to pulsars born with luminosities above 10 mJy kpc\(^2\). All previous estimates applied to pulsars born with luminosities above \( \sim 0.3 \) mJy kpc\(^2\) which, as we have demonstrated, are not present in sufficient numbers to derive their birthrate accurately. To be consistent with Clark & Stephenson (1977), we would require more than three times as many pulsars to be born with luminosities below 10 mJy kpc\(^2\) than above it. None of the eight pulsars currently known to be associated with supernova remnants has a luminosity below \( \sim 50 \) mJy kpc\(^2\), assuming distances derived from the Cordes et al. (1991) distance model. However, a careful analysis of the selection effects governing searches for pulsars in supernova remnants needs to be undertaken in
order to discover whether this is solely due to selection effects, or is further evidence that very few pulsars are born with low luminosities.

5 CONCLUSIONS
In this paper, we have used Monte Carlo simulations of the galactic pulsar population to test previous approaches and to develop a new method of determining the birthrate and initial spin periods of radio pulsars. Our results are as follows.

(i) The basic pulsar-current analysis used by VN is statistically flawed for the prediction of the initial spin periods of pulsar populations, being biased by a small number of low-luminosity nearby pulsars and the large uncertainties in their distances. This effect is demonstrated by tests on plausible models of the pulsar population, whose birth properties and luminosities are well understood.

(ii) Previous attempts to improve the statistical significance of the analysis by VN and Narayan (1987) by using a luminosity model may have introduced into the analysis a systematic bias against short-period pulsars, thus inducing injection.

(iii) Neither the luminosity law proposed by Gunn & Ostriker (1970) and used by LMT in their calculations, nor the law used by Narayan (1987), adequately describes the observed luminosities of pulsars.

(iv) By excluding low-luminosity pulsars in a pulsar-current analysis, we can greatly improve the method, allowing us to make accurate predictions about the birthrates and initial spin periods of model populations. Applying this luminosity exclusion technique to the observed pulsar sample, we conclude that there is little evidence for any pulsars being born as slow rotators.

(v) We estimate the birthrate of all pulsars in the Galaxy with luminosities above 10 mJy kpc\(^2\) to be (0.60 ± 0.20) pulsar century\(^{-1}\). The total number of potentially observable pulsars with luminosities above 10 mJy kpc\(^2\) is estimated to be (1.31 ± 0.17) × 10\(^4\). The local birthrate of pulsars in the solar neighbourhood is estimated to be between 6 and 12 pulsar kpc\(^{-2}\) Myr\(^{-1}\). This is in excellent agreement with the estimate by Blaauw (1987) of the local production rate of OB stars.

(vi) Our galactic pulsar birthrate estimate is consistent with the supernova remnant rate derived by Clark & Caswell (1976). To be consistent with larger estimates (e.g. Clark & Stephenson 1977), we find that the majority of pulsars would have to be born with luminosities below 10 mJy kpc\(^2\). As yet there is no observational evidence that this is the case; we have found only marginal evidence that pulsars are born with luminosities below 30 mJy kpc\(^2\).

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