Further results from the timing of the millisecond pulsars in 47 Tucanae

P. C. Freire, F. Camilo, M. Kramer, D. R. Lorimer, A. G. Lyne, R. N. Manchester and N. D’Amico

Abstract

We have been observing the millisecond pulsars in the globular cluster 47 Tucanae (47 Tuc) at the Parkes radio telescope since 1999 August with threefold higher time-resolution than hitherto possible. We present the results in this paper, including: improved 1400-MHz pulse profiles; one new timing solution, for PSR J0024−7204S, which imposes stringent constraints on the acceleration model for 47 Tucanae and implies a projected mass-to-light ratio \( > 1.4 \, M_\odot / L_\odot \) at the centre of the cluster; refined estimates for the five previously determined proper motions; and newly determined proper motions for six pulsars. We have detected, for the first time, relative motions between the pulsars. We have detected a second period derivative for the pulsar in the PSR J0024−7204H binary system, which could indicate the presence of a third nearby object, and improved measurement of the rate of advance of periastron of this pulsar, which yields a total system mass of \( 1.61 \pm 0.04 \, M_\odot \). We also have determined upper limits for the masses of any hypothetical planets orbiting the pulsars in 47 Tuc. PSR J0023−7203J shows variations of dispersion measure (DM) as a function of orbital phase with a total column density at superior conjunction of about \( 1.7 \times 10^{16} \, \text{cm}^{-2} \), 10 times smaller than observed for a similar system in the Galaxy. We interpret the small value as being due to a smaller inclination of the orbit of PSR J0023−7203J. We find that the DM variation with orbital phase changes with time, and we detect material at more than 90° (in orbital phase) from the companion. PSR J0024−7204O also shows variations of DM with orbital phase, but these are restricted to phases near the eclipse. This binary system displays significant monotonic variation of its orbital period: \( \dot{P}_b = (9 \pm 1) \times 10^{-12} \). This is probably due to spin–orbit coupling; this effect seems to be significantly smaller for PSR J0023−7203J.

Key words: binaries: general – stars: neutron – pulsars: general – globular clusters: individual: 47 Tucanae.

1 Introduction

During the last 12 years, 20 pulsars have been discovered in the globular cluster 47 Tucanae (Manchester et al. 1991; Robinson et al. 1995; Camilo et al. 2000). For 15 of these pulsars, phase-coherent timing solutions have been previously published (Robinson et al. 1995; Camilo et al. 2000; Freire et al. 2001a, henceforth Paper I) and in this paper we provide a further one, for PSR J0024−7204S (hereafter we use shortened names for pulsars, 47 Tuc S in this case).

Between 1999 August and 2002 March, we have been observing 47 Tucanae (henceforth 47 Tuc) using the 2 × 512 × 0.5 MHz filter bank at the Parkes radio telescope. The narrow channels and the low dispersion measure (DM) of the pulsars in the cluster produce a dispersive smearing of 37 \( \mu \text{s} \) across a single channel at the central frequency of 1390 MHz. We use the shortest sampling time allowed by the data acquisition system, 80 \( \mu \text{s} \). The resulting time-resolution of 88 \( \mu \text{s} \) represents a threefold improvement over previous 1400-MHz data (Camilo et al. 2000).

Among the more immediate benefits of observations of pulsars with a high time-resolution is a more precise measurement of pulse times of arrival (TOAs) which, in turn, leads to an improvement in all timing parameter determinations. Using these data to better...
estimate DMs, we have recently demonstrated that the pulsars on the far side of 47 Tuc have, on average, higher DMs than those on the near side (Freire et al. 2001b). The gradient in DM with radial position in the cluster indicates the presence in the cluster of plasma with a free-electron density of \( (0.067 \pm 0.015) \, \text{cm}^{-3} \). An additional benefit from high time-resolution is the improvement of the pulse profiles. For the 16 pulsars with coherent timing solutions, these are presented in Fig. 1. These have some features still unresolved (e.g. in the two main pulses of 47 Tuc E and the profiles of 47 Tuc J and O), and also features never seen before, such as the precursor for 47 Tuc C, the interpulse for 47 Tuc D, which lies at a phase of about 0.95 in Fig. 1, and the many features in the complex pulse profiles of 47 Tuc G and T. The latter displays an interpulse and a sharp feature immediately after the main pulse. For other pulsars with complex pulse profiles, such as 47 Tuc L, M, Q and U, we can now observe the profiles far more clearly than possible before. The pulse width of 47 Tuc S is similar to the time-resolution of the observing system.

In Section 3 we discuss the positions and proper motions obtained from the timing programme, and in Section 4 we analyse the rotational parameters and their derivatives. This includes a discussion of the period derivative of 47 Tuc S and the constraints it places on cluster parameters. We also present the detection of a second period derivative for 47 Tuc H, plus an investigation of the existence of planets among the 16 pulsars with timing solutions. In Section 5 we analyse the binary parameters for 47 Tuc H. This pulsar now has an improved measurement of the rate of advance of periastron; we use.
it to calculate the probability distribution for the mass of the pulsar and its companion. In Section 6 we discuss the mass distribution of the pulsar companions observed in the Galaxy and in clusters, and the mass distribution of very low-mass binary pulsars in particular. In Section 7 we describe in the detail made possible by the data the eclipse properties of 47 Tuc J and O. In Section 8 we address their orbital evolution and compare them with similar Galactic systems. We conclude in Section 9 with a summary of the results.

2 THE TIMING SOLUTIONS

In Tables 1 and 2, we present the full timing parameters for 16 millisecond pulsars in 47 Tuc obtained with the timing analysis program TEMPO. 15 of these (all except 47 Tuc S) had phase-coherent timing solutions published previously in Paper I. Here we use the same data used in the previous analysis (i.e. data obtained since 1991 May) and we include 31 months of the new high-resolution data (from 1999 August to 2002 March).

Because of the difference in time-resolution between these and the earlier 1400-MHz data, we must use different pulse profile templates to measure the TOAs for the high-resolution data. These templates were derived from the best detection of each pulsar at this frequency and time-resolution, both by smoothing or/isolating the main pulse. The use of different templates for the different data sets introduces an astrophysically meaningless time jump between the two sets of 1400-MHz TOAs, which we fit for. The clock system used, its corrections, the Solar system ephemeris and the methods used to derive and correct the TOAs are as described in Paper I. For pulsars 47 Tuc J and O, which show variations of DM with orbital phase, we describe additional methods used to obtain the timing solutions in Section 4. For 47 Tuc H we have used the Damour–Deruelle (DD) orbital model (Damour & Deruelle 1986), while for the remaining binary systems we used the ELLI1 orbital model (Lange et al. 2001).

The values presented in Tables 1 and 2 are derived from a straightforward TEMPO fit, which includes all the indicated timing parameters. The DMs are the same as in Freire et al. (2001b), except for the cases of 47 Tuc J and O (see Section 7). For all pulsars, with the exception of 47 Tuc H, the second time derivative of the spin frequency $\dot{\nu}$ was also found to be consistent with zero, so we assumed it to be so while deriving the remaining parameters. The values and upper limits for this parameter are presented in Table 3. In Section 3.2 we describe how the proper motions were handled. For the binaries 47 Tuc J and O, the eccentricity was found to be consistent with zero, so we assumed zero orbital eccentricity while deriving the remaining parameters. The upper limits indicated are three times the 1σ uncertainty obtained for that parameter when it is fitted together with all the non-zero parameters. For all pulsars, with the exception of 47 Tuc J and O (Section 8), we assume non-existent orbital period derivatives.

The timing parameters measured are mostly consistent at the 2σ level with those reported in Paper I. Exceptions are the determinations of $P$ and $\dot{P}$ for 47 Tuc H, which are affected by the fit of $\dot{\nu}$, and the orbital parameters for 47 Tuc J and O, which are significantly modified by the DM corrections described in Section 7 and the fitting of the orbital period derivatives described in Section 8.

Finally, we present an entirely new solution for 47 Tuc S, a 2.83-ms pulsar in a 1.2-d binary system (Camilo et al. 2000; Freire, Kramer & Lyne 2001) with a significant measurement of eccentricity. This timing solution is characterized by a very large negative

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Table 2. Timing parameters for nine binary pulsars in the globular cluster 47 Tuc. Timing fit and pulsar parameters are as in Table 1. The orbital parameters are the orbital period \( P \), the projected semimajor axis of the pulsar orbit around the centre of mass \( a \), the time of ascending node \( T_{\text{asc}} \), except for 47 Tuc H, for which this quantity indicates the time of passage through periastron, the longitude of periastron \( \omega \), the eccentricity \( e \) and the rate of advance of periastron \( \dot{\omega} \). Derived parameters are the pulsar mass function \( f \) and the minimum companion mass \( m_c \), calculated assuming that the mass of the pulsar is precisely 1.35 \( M_\odot \). Upper limits for \( e \) are 3\% values.

<table>
<thead>
<tr>
<th>Pulsar (J2000)</th>
<th>0024−7203E</th>
<th>0024−7204H</th>
<th>0024−7204I</th>
<th>0023−7203J</th>
<th>0024−7204O</th>
<th>0024−7204Q</th>
<th>0024−7204S</th>
<th>0024−7204T</th>
<th>0024−7203U</th>
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<td>Start MJD</td>
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<td>49817</td>
<td>50683</td>
<td>49832</td>
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<td>50689</td>
<td>50689</td>
<td>52349</td>
<td>52349</td>
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<tr>
<td>Final MJD</td>
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<td>52550</td>
<td>52347</td>
<td>52535</td>
<td>52357</td>
<td>52349</td>
<td>52686</td>
<td>52357</td>
<td>52349</td>
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<tr>
<td>NTTOs</td>
<td>1184</td>
<td>512</td>
<td>296</td>
<td>123</td>
<td>296</td>
<td>296</td>
<td>169</td>
<td>138</td>
<td>149</td>
</tr>
<tr>
<td>rms (( \mu ))</td>
<td>9</td>
<td>20</td>
<td>14</td>
<td>14</td>
<td>3</td>
<td>3</td>
<td>11</td>
<td>49</td>
<td>9</td>
</tr>
<tr>
<td>( \alpha ) (( \text{mas} ))</td>
<td>0.0241111066 (1)</td>
<td>0.0240670143 (3)</td>
<td>0.0240739930 (3)</td>
<td>0.0235940753 (3)</td>
<td>0.024046512 (4)</td>
<td>0.024164891 (4)</td>
<td>0.024139717 (4)</td>
<td>0.024132442 (4)</td>
<td>0.024100549 (2)</td>
</tr>
<tr>
<td>( \dot{\omega} ) (( \text{mas} ))</td>
<td>6.2 (1)</td>
<td>5.1 (2)</td>
<td>4.9 (16)</td>
<td>5.3 (3)</td>
<td>4.6 (11)</td>
<td>4.6 (11)</td>
<td>4.3 (2)</td>
<td>4.1 (2)</td>
<td>4.0 (2)</td>
</tr>
<tr>
<td>( \mu_x ) (( \text{mas} ))</td>
<td>-2.7 (5)</td>
<td>-3.6 (12)</td>
<td>-3.9 (19)</td>
<td>-3.4 (15)</td>
<td>-2.9 (9)</td>
<td>-1.4 (2.2)</td>
<td>-5.4 (2.8)</td>
<td>-5.1 (7)</td>
<td>-5.1 (7)</td>
</tr>
<tr>
<td>( \nu ) (( \text{cycles} ))</td>
<td>282.779093543175 (5)</td>
<td>311.945349744424 (4)</td>
<td>286.94469993106 (4)</td>
<td>476.46085840070 (6)</td>
<td>378.30788893075 (7)</td>
<td>247.94332741680 (3)</td>
<td>353.30602939559 (5)</td>
<td>131.77609474127 (2)</td>
<td>230.2641722192 (2)</td>
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<td>( \dot{\nu} ) (( \text{cycles} ))</td>
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<td>0.178 (1)</td>
<td>0.377 (1)</td>
<td>3.434 (1)</td>
<td>-2.091 (1)</td>
<td>-2.091 (1)</td>
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<td>-2.091 (1)</td>
<td>-2.091 (1)</td>
</tr>
<tr>
<td>( P_1 ) (( \text{ms} ))</td>
<td>3.53603291576302 (5)</td>
<td>3.21032404043844 (4)</td>
<td>3.48499300666151 (3)</td>
<td>2.1063343572147 (2)</td>
<td>2.643433279417 (2)</td>
<td>2.830405957877 (2)</td>
<td>2.75884790736 (4)</td>
<td>3.4328626693826 (4)</td>
<td></td>
</tr>
<tr>
<td>( v ) (( \text{cm} ))</td>
<td>9.85 (10)</td>
<td>-0.183 (1)</td>
<td>-0.979 (29)</td>
<td>0.0354 (9)</td>
<td>3.402 (2)</td>
<td>-12.054 (2)</td>
<td>29.37 (1)</td>
<td>9.523 (1)</td>
<td></td>
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<td>( \mu_0 ) (( \text{cm} ))</td>
<td>2.25604820 (8)</td>
<td>2.37536868 (8)</td>
<td>0.229702942 (9)</td>
<td>0.126069779 (4)</td>
<td>0.1136713805 (4)</td>
<td>1.1809840848 (2)</td>
<td>1.2017242356 (4)</td>
<td>0.4291053833 (2)</td>
<td></td>
</tr>
<tr>
<td>( \tau ) (( \text{pc} ))</td>
<td>0.0013522 (1)</td>
<td>0.0070560 (3)</td>
<td>&lt;0.0000036</td>
<td>&lt;0.000004</td>
<td>&lt;0.000016</td>
<td>&lt;0.0000055 (5)</td>
<td>0.0000394 (7)</td>
<td>0.000040 (2)</td>
<td>0.00004194 (4)</td>
</tr>
<tr>
<td>( f ) (( \text{Hz} ))</td>
<td>0.0016441</td>
<td>0.00626</td>
<td>0.000001156</td>
<td>0.000000864</td>
<td>0.000000345</td>
<td>0.000000345</td>
<td>0.000000345</td>
<td>0.000000345</td>
<td>0.000000345</td>
</tr>
<tr>
<td>( m_c ) (( \text{M}_\odot ))</td>
<td>&gt;0.15</td>
<td>&gt;0.16</td>
<td>&gt;0.16</td>
<td>&gt;0.16</td>
<td>&gt;0.16</td>
<td>&gt;0.16</td>
<td>&gt;0.16</td>
<td>&gt;0.16</td>
<td>&gt;0.16</td>
</tr>
</tbody>
</table>

3.1 Positional coincidences

The timing solution of 47 Tuc indicates that, in projection, the pulsars lie only about 0.1 arcsec from the cluster centre. There are six globular clusters containing two or more pulsars, which can be considered to be close, and yet further searching may reveal another two or more clusters. The probability of finding a pair of pulsars close together in our simulation is 7\%\( \times 0.1 \), which is close to the expected value of 6\%\( \times 0.1 \) in the Tucana-Hercules region of the sky. The observed positional distribution of the pulsars is consistent with the PSF, which places strong constraints on the cluster parameters (see Section 4).
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(Biggs et al. 1994) and NGC 6752 (D’Amico et al. 2002) (see http://www.naic.edu/~pfreire/GCpsr.html). What is the probability of finding one (or more) pairs of objects in the ensemble of these clusters that has the same statistical unlikeliness of the 47 Tuc G-I pair? The answer is 1 − (1 − 7 × 10−4)6 = 0.0042. From this point of view, finding something like the 47 Tuc G-I pair in the ensemble of six globular clusters is not even a 3σ event, i.e., although rare, it is of no particular statistical significance. The probability of finding a pair like 47 Tuc F-S in the ensemble is about 20 per cent.

If the pulsars in each pair were gravitationally bound, the distances between the members of these pairs (at least 600 a.u. for G-I and 3700 a.u. for F-S) would imply orbital periods greater than the typical survival time of the hypothetical system in the dense environment of the globular cluster (see Paper I). This makes such systems unlikely.

However, we can now test the nature of the association more directly. The hypothetical orbital motion, and the resulting change in the acceleration vector, should cause a variation in \( \dot{v} \) (Section 4.2), given by equation (2). The effects on \( \dot{v} \) that could be caused by the cluster and the observational limits are presented in Table 3. The limits are smaller for 47 Tuc G, so we calculate the effect of 47 Tuc G on this pulsar. Assuming that the separation of 47 Tuc G and I is \( \sqrt{2} \Delta R \) (\( \Delta R \) is the projected separation between the two pulsars, which at the 5-kpc distance of 47 Tuc is \( \times 10^{15} \) cm), and that the mass of the 47 Tuc I binary system is about 1.4 M\(_{\odot}\) (which is reasonable, as the minimum companion mass for that system is about 0.013 M\(_{\odot}\)) we obtain an upper limit for the effect of 47 Tuc I on G:

\[
|v| \lesssim \frac{G M_1}{\sigma_{v G} b_1} \Rightarrow \dot{v} \lesssim 0.9 \times 10^{-25} \text{ s}^{-3}.
\]

Here \( v \) is of the order of the expected orbital velocity (\( \sqrt{G M_1/b_1} \sim 1.2 \text{ km s}^{-1} \)). This is larger than the prediction for \( \dot{v} \) caused by the cluster (Table 3). The inequality is comparable with the present observational upper limit for \( |v| \), which is \( 8.8 \times 10^{-25} \text{ s}^{-3} \) (3σ).

To summarize, there are as yet no strong arguments (statistical or otherwise) to suggest that the 47 Tuc G-I pair is more than a chance alignment. Because of the presence of plasma in the cluster (Freire et al. 2001b), the similar DMs suggest that the pulsars lie close to each other in the third dimension as well, but nothing indicates that they are gravitationally bound.

### 3.2 Proper Motions

The precision of the five previously measured proper motion values has improved with the use of high-resolution timing data, as can be seen in Fig. 2. The proper motion of 47 Tuc D is incompatible with the proper motions of 47 Tuc J and E. The difference of the motion of 47 Tuc D and 47 Tuc J and E amounts to \((1.8 \pm 0.4)\) and \((2.4 \pm 0.6)\) mas yr\(^{-1}\), respectively. Assuming a distance of 5 kpc, this corresponds to relative transverse velocities of \((44 \pm 10)\) and \((57 \pm 14)\) km s\(^{-1}\), respectively. These values are similar to the escape velocity of the cluster, 58 km s\(^{-1}\) (Webbink 1985). However, the 3σ uncertainties of the differences of the transverse motions are still nearly as large as the motions themselves, so at this moment it is effectively impossible to reach any definitive conclusions about what exactly is happening. The important point to retain is that we have apparently detected, for the first time, intrinsic motions of the pulsars relative to each other.

We have been able to obtain, for the first time, significant (3σ) measurements of proper motion for the pulsars 47 Tuc G, H, I, N, O and U, which are indicated in Tables 1 and 2. For the other pulsars, the composite proper motions are still consistent with zero; their nominal values are tabulated in square brackets. For these pulsars, the average motion of the cluster (\( \mu_\alpha = [5.3 \pm 0.6] \) and \( \mu_\delta = [-3.3 \pm 0.6] \) mas yr\(^{-1}\)) was adopted when fitting for the remaining timing parameters.

This value for the average motion of the cluster was estimated by the weighted average of all the significant pulsar proper motions. We have excluded the motion of 47 Tuc D from this average, because of its seemingly large peculiar velocity. The difference between the nominal values for the motion of the cluster obtained using Hipparcos and using the pulsars is not significant. It is also not due to the choice of reference ephemeris; using the new JPL DE 405 planetary ephemeris, we obtain differences of proper motion of the order of 0.1 mas yr\(^{-1}\).

### 4 ROTATIONAL PARAMETERS

#### 4.1 The period derivative of 47 Tuc S

The observed \( P/P \) for 47 Tuc S (\( -4.30 \times 10^{-17} \text{ s}^{-1} \)) implies for this pulsar the largest line-of-sight acceleration measured so far for a pulsar in 47 Tuc: \( |\alpha| > c |P/P| = 1.3 \times 10^{-6} \text{ cm s}^{-2} \). As discussed in Paper I, this acceleration, if due to the cluster, allows us to constrain the cluster parameters.

Are there other possible causes for the large \( |\alpha| \)? It is possible, given the high stellar density typical of the central regions of a globular cluster, that the gravitational field of a nearby star is accelerating 47 Tuc S to a larger extent than the cluster. As shown by Phinney (1993, Section 4.1) the probability of this happening is \( \sim 0.2 N^{-1/2} \), where \( N \) is the number of stars in the core of the cluster. Assuming \( N \sim 10^5 \) (Pryor & Meylan 1993), we find the probability to be only 0.0006. We know that, in projection, there is a star near 47 Tuc S, namely 47 Tuc F. However, the maximum line-of-sight acceleration
that 47 Tuc F can produce in any point along the line of sight of 47 Tuc S is 40 times smaller than \(|a_S|\).

The acceleration of 47 Tuc S is therefore likely to be due to the gravitational field of the globular cluster. According to the arguments presented in section 5.1 of Paper I, we can derive from it a lower limit for the surface mass density of the cluster within its projected distance from the centre of the cluster (11.7 arcsec): 8.4 \(\times\) 10\(^{2}\) M\(_{\odot}\) pc\(^{-2}\). The luminosity density within 11.7 arcsec is \(\sim\) 6 \(\times\) 10\(^{5}\) L\(_{\odot}\) pc\(^{-2}\) (Gebhardt & Fischer 1995), therefore \(M/L > 1.4 M_{\odot}/L_{\odot}\) for this region. This is consistent with the values obtained by Gebhardt & Fischer (1995) for the central \(M/L\) of 47 Tuc.

The maximum acceleration along any line of sight, \(a_{\text{max}}\), occurs along that which passes through the centre of the cluster. To \(\sim\)10 per cent accuracy, Phinney (1993) has shown that

\[
a_{\text{max}} = 3/2 \frac{v_2(0)}{D\theta_c},
\]

where \(v_2(0)\) is the line-of-sight stellar velocity dispersion at the centre of the cluster and \(D\) is the distance of the cluster from the Sun. The acceleration of any pulsar must be less than this value (see Paper I), so that the measured value for \(a_S\) introduces a strong constraint on \(v_2(0)/D\). This can be seen in Fig. 3 where we have taken the core radius \(\theta_c\) to be 23.1 arcsec (Howell, Guhathakurta & Gilliland 2000). The cluster parameters as given by the references presented in section 5.1 of Paper I, we can derive from it a lower limit for the gravitational acceleration at the particular line of sight of 47 Tuc S, we obtain a model-independent upper limit for its intrinsic \(\dot{P}/P\) of 4.1 \(\times\) 10\(^{-2}\). From this, we estimate a minimum characteristic age \(t_\text{c} = (\dot{P}/2P)\) of 1.1 Gyr and a maximum magnetic field at the neutron star surface \(B=(3.2 \times 10^9 \sqrt{P/\dot{P}})\) of 3.4 \(\times\) 10\(^3\). Using a King model to estimate the maximum possible acceleration at the particular line of sight of 47 Tuc S, we obtain more stringent values: \(\dot{P} < 1.7 \times 10^{-20}\), \(t_\text{c} > 2.6\) Gyr and \(B < 2.2 \times 10^8\) (see Paper I for details).

4.2 The second frequency derivative of 47 Tuc H

Using the astrometric, spin (\(v\) and \(\dot{v}\)) and binary parameters, we cannot ‘predict’ correctly the pulse TOAs of 47 Tuc H in the early 1990s, even with complete freedom in the fitting of the time jump between the early and late 1990s. In Fig. 4 we show the residuals for 47 Tuc C and H; the former pulsar (and indeed, all others in the cluster) do not show the same trend in the early 1990s, as they should if the cause of the anomaly for 47 Tuc H were due to clock or measurement error.

A potential explanation would be a secular variation in the DM of 47 Tuc H. A DM variation of \(-0.024(3)\) cm\(^{-3}\) pc yr\(^{-1}\) eliminates the trend observed for the 430-MHz residuals. This value of \(d(\text{DM})/d\tau\) is similar to the largest DM variations measured for any pulsar (Hobbs 2002; Splaver et al. 2002), but no other pulsar in 47 Tuc shows similar behaviour; therefore such an explanation is unlikely. Also, no binary parameter available can explain the observed trend.

Fitting for a second derivative of the rotational frequency also eliminates the trends observed in the early 1990s, leaving no structure in the residuals (see Fig. 4). The value of \(\ddot{v}\) is quite significant: \((1.6 \pm 0.2) \times 10^{-25}\) s\(^{-3}\). Making such a fit for \(\ddot{v}\) using only the 1400-MHz TOAs we obtain \((2.1 \pm 0.8) \times 10^{-25}\) s\(^{-3}\). This is as large as the effect measured using the whole data set, which suggests that the observed derivative of \(v\) is independent of frequency, and therefore not likely to be a propagation effect.

The effect could be due to timing noise. With two exceptions – PSRs B1937+21 (Kaspi, Taylor & Ryba 1994), and possibly J1012+5307 (Lange et al. 2001) – millisecond pulsars display no timing noise measured at the level of precision used to study them, even when the precision is much higher than for 47 Tuc H. For PSRs B1937+21 and J1012+5307, the measured \(\dddot{v}\) are \(-13.2(0.3) \times 10^{-27}\) and \(-9.8(2.1) \times 10^{-27}\) s\(^{-3}\), respectively, an order of magnitude smaller than what is observed for 47 Tuc H. Therefore, it is unlikely that timing noise is responsible for the large observed \(\ddot{v}\) of this pulsar.

Another possible cause of \(\ddot{v}\) is the acceleration of the pulsar in the cluster potential. According to Phinney (1993), a gravitationally-induced second period derivative is given, to good approximation, by

\[
|\ddot{v}| P \equiv \left| \frac{\dot{P}}{P} \right| \equiv \sum_i G M_i |v_i| b_i^{-6} \frac{c b_i}{R_i},
\]

where \(M_i\) and \(b_i\) are the masses and distances to the pulsar of all objects in the cluster. Equating \(v\) with the maximum possible line-of-sight stellar velocity dispersion at the centre of the cluster (\(~13.0\) km s\(^{-1}\); Meylan & Mayoral 1986), we obtain an upper limit for \(\ddot{v}\) due to the cluster:

\[
|v_c| < \frac{G M(r) |v|}{P c r^2} \leq \frac{a_{\text{max}}(R_c)}{P c} \frac{v_2(0)}{\sqrt{2} R_c}.
\]
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Figure 4. Residuals as a function of time for 47 Tuc C and H. Top plot: residuals for 47 Tuc C using the best-fitting \( \nu \) and \( \dot{\nu} \), offset by +800 \( \mu s \). Middle plot: residuals for 47 Tuc H using the best-fitting \( \nu \) and \( \dot{\nu} \). Bottom plot: residuals obtained for 47 Tuc H when \( \ddot{\nu} \) is also fitted for, offset by \( -800 \mu s \). The \( \times \) symbols indicate low-resolution 1400-MHz residuals, the + symbols indicate high-resolution 1400-MHz residuals, the triangles indicate 430-MHz residuals and the squares denote 660-MHz residuals.

The values for \( a_{\text{max}}/c \) are taken from Paper I. The factor \( \sqrt{2} R_\perp \) is the distance from the centre \( r \) (along the line of sight that passes at a distance \( R_\perp \) from the centre) for which we observe the maximum line-of-sight acceleration. Table 3 compares \( \ddot{\nu} \) along the line of sight of each pulsar to the observed \( \ddot{\nu} \), or its measured upper limit. For some pulsars (47 Tuc F and O), \( \ddot{\nu} \) is now only slightly smaller than the uncertainty in \( \ddot{\nu} \). For most of the other pulsars \( \ddot{\nu} \) is much smaller than the uncertainty in the measurement. The observed \( \ddot{\nu} \) for 47 Tuc H is most clearly not caused by the gravitational field of the cluster.

The list of viable causes of the effect include: (a) a moderately close encounter with another star (this is possible in the dense environment of the globular cluster); (b) the binary belonging to a wide hierarchical system; and (c) the binary having a planetary companion. Possibility (c) is discussed in the following section.

We remark that 47 Tuc H is also the binary pulsar with the highest known eccentricity in 47 Tuc. We have at present no information to conclude whether these two facts are causally related, or if they just happen to be coincident.

4.3 A search for planets around the 47 Tuc pulsars

Davies & Sigurdsson (2001) have studied in detail the stability of planets in 47 Tuc. They find that planetary orbits with semimajor axes larger than 0.3 a.u. are likely to be disrupted in the lifetime of the cluster if the parent star lies (as the pulsars do, at least in projection) within the 3.9-pc half-mass radius of the cluster (Howell et al. 2000). Therefore, a planet with an orbital period of many decades is an unlikely explanation for the \( \ddot{\nu} \) of 47 Tuc H. Tighter planetary systems, or systems at larger distances from the centre of the cluster, are likely to survive.

A search for planets transiting in front of main-sequence stars in 47 Tuc has been undertaken with the Hubble Space Telescope (HST) (Gilliland et al. 2000). No such systems were found, when \( \sim ~17 \) would have been expected if the incidence of planetary systems in 47 Tuc were the same as for the disc of the Galaxy.

A search with sensitivity to much lighter companions around the known pulsars can be made by analysing the pulsar timing residuals for unmodelled periodic signals. We adopted the approach described by Bell et al. (1997), calculating the Lomb–Scargle spectra of pulse TOA residuals. Their algorithms detected the presence of planets around PSR B1257+12 (Wolszczan & Frail 1992; Wolszczan 1994), but failed to detect any periodic oscillations from any of the other millisecond pulsars surveyed, none of which is known to possess planetary companions.

Our calculation of the Lomb–Scargle spectra and the limits for a significant detection are slightly different from that of previous authors, as described in the appendix. The results can be seen in Fig. 5 for 47 Tuc H. There are no highly significant periodicities in the residuals of any pulsar we surveyed, to the rather stringent mass limits indicated in the plots. For instance, for 47 Tuc J, we can exclude the presence of 0.002-Earth-mass planets with orbital periods of a year. We used the same procedure for the residuals obtained assuming \( \ddot{\nu} = 0 \) for 47 Tuc H. There is a marginally significant periodicity in this case, but the period (1600 d) is larger than the time period covered by the 1400-MHz observations. Fitting

2 For the spectra of the remaining pulsars, see http://www.jb.man.ac.uk/~pulsar/47Tuc.html.
Figure 5. Lomb–Scargle spectra for 47 Tuc H. In each plot, the upper solid, dashed and dotted lines indicate the 99.9, 99.5 and 95 per cent confidence limits, respectively. No significant periodicities are observed in the spectrum of 47 Tuc H, either with (lower plot) or without (upper plot) a fit for $\nu$ before obtaining the spectra.

for $\nu$, this periodicity, and in fact almost all of the power at low frequencies, disappears completely, indicating the secular nature (at least compared to the observation span) of $\nu$.

5 THE MASS OF THE 47 TUC H BINARY SYSTEM

The measurement accuracy of the rate of advance of periastron of the 47 Tuc H binary system ($\dot{\omega} = 0.066\pm0.002$ yr$^{-1}$) has improved by a factor of 4 over that quoted in Paper I ($\dot{\omega} = 0.055\pm0.008$ yr$^{-1}$). These two uncertainties were obtained by multiplying the formal TEMPO uncertainty by a factor of 2. An alternative way of estimating this uncertainty is by performing a Monte Carlo bootstrap calculation. Anderson et al. (1997) estimated the $\dot{\omega}$ of PSR B1516+02B using the bootstrap method described in Efron & Tibshirani (1993). A similar calculation for 47 Tuc H with 4096 iterations yields $\dot{\omega} = (0.066 \pm 0.001)$ yr$^{-1}$.

Splaver et al. (2002) have recently studied the PSR J0621+1002 binary system. This system also has a well-measured $\dot{\omega}$, but lacks a detectable measurement of the Shapiro delay, which indicates a low inclination relative to the plane of the sky. This fact can be used to impose constraints on the masses of the pulsar and its companion. For each point on a grid of possible inclinations and companion masses they calculate the rate of advance of periastron and the Shapiro delay predicted by general relativity for the sum of the masses and the known orbital parameters. $\dot{\omega}$, $i$, and $m_\odot$ are kept constant during the TEMPO fit used to obtain the best possible timing solution consistent with these parameters. The resultant $\chi^2$ is then used to calculate a probability density for the set of relativistic parameters, using the Bayesian techniques described in the appendix of Splaver et al. (2002).

Applying this technique to the TOAs of 47 Tuc H, we obtain for $\dot{\omega}$ a perfect Gaussian distribution, characterized by an average of 0.0658 yr$^{-1}$ and a standard deviation of 0.0009 yr$^{-1}$. This allows a good estimate of the total mass of the system: $(1.61 \pm 0.04) M_\odot$. The quality of the fit does not change considerably with the inclination of the system, but it is best for $90^\circ$ ($\chi^2 = 497.97$), decreasing monotonically towards $0^\circ$ ($\chi^2 = 500.41$).

Taking the probability density function (p.d.f.) of $i$, $p(i) = \exp(\chi^2(i))/\chi^2(i)/2$, and the Gaussian p.d.f. of $\dot{\omega}$ into account, we can calculate a two-dimensional (2D) p.d.f. for the position of the system in the mass–mass diagram. The three contour lines in Fig. 6 enclose 68.26, 95.44 and 99.74 per cent of all probability. The vertical lines correspond to the average and r.m.s. of the available values for neutron star masses reported in Thorsett & Chakrabarty (1999): $1.35 \pm 0.04 M_\odot$. They are displayed for comparison purposes.

We projected the 2D p.d.f. upon the two axes to derive individual one-dimensional (1D) p.d.f.s for the pulsar and companion mass. The latter is strongly peaked at around a median of $0.18 M_\odot$, and then displays a long tail towards higher masses; this tail is smaller than it would be for a random inclination distribution. The 68 per cent integrated probability around the median ranges from 0.164 to 0.266 $M_\odot$. The p.d.f. for the pulsar mass is broader: the median is $1.41 M_\odot$ and the 68 per cent integrated probability about the median extends from 1.33 $M_\odot$ to 1.45 $M_\odot$. The p.d.f.s show that $m_p < 1.52 M_\odot$ and $m_c > 0.164 M_\odot$.

In all these calculations, we have assumed that the observed $\dot{\omega}$ is due entirely to general relativity, with a negligible amount due to classical effects. Smarr & Blandford (1976) have determined that tidal deformations would not be important if the companion of PSR B1913+16 was a heavy white dwarf; this effect is much smaller for 47 Tuc H as it scales with $a^{-3}$, where $a$ is the orbital separation. The rotational deformation, due to the rotation of the companion, might provide an important contribution to $\dot{\omega}$ if the companion were rotating rapidly. However, we have no reason to expect rapid rotation of the companion, and even in this event a
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Figure 6. Mass–mass diagram for the 47 Tuc H system. The allowed range of total mass (slowing straight line surrounded by dashed lines) is derived from the measured rate of advance of periastron and its $3\sigma$ uncertainty. The area below the thick curving line is excluded by the measured mass function and the requirement that $\cos i \geq 0$. The vertical lines indicate the average measured mass for neutron stars in the radio pulsar population (Thorsett & Chakrabarty 1999). The contour levels apply to the system’s 2D p.d.f., and include 68.3, 95.4 and 99.74 per cent of probability. Also plotted are the p.d.f.s for the mass of the pulsar and the mass of the companion.

special spin–orbit geometry would be required to make this effect significant (see Splaver et al. 2002), so we consider it likely that the classical contribution to $\dot{\omega}$ is negligible.

6 VERY LOW-MASS BINARY PULSARS

47 Tuc I, J, O, P and R have $\sim 0.02 M_\odot$ companions (Table 2; Camilo et al. 2000) and short orbital periods. These binaries form a population that is distinct from that of the more common neutron-star–white-dwarf binaries, here referred to as low-mass binary pulsars (LMBPs). We will henceforth call these very low-mass binary pulsars (VLMBPs). The difference in companion masses can clearly be seen in Fig. 7, and is expected from simulations of binary evolution (Davies & Hansen 1998; Rasio, Pfahl & Rappaport 2000). This plot contains 12 of the 13 known eclipsing binary pulsars, the exception being PSR B1259–63, which has a Be star as companion (Johnston et al. 1992) and a 3.2-yr binary period.

The two binary pulsars in 47 Tuc with the lowest masses, 47 Tuc I and P, do not display eclipses, despite having orbital periods similar to those of the eclipsing VLMBPs, 47 Tuc J, O and R. 47 Tuc P is similar in many respects to PSR B1908+00 in NGC 6760 (Deich et al. 1993); during their detections (all at 1400 MHz, of which there is only one for 47 Tuc P) none of these pulsars displayed eclipses. For 47 Tuc I, the pulsar with the (apparently) lightest companion in 47 Tuc, the 1400-MHz TOAs are well described by a circular orbit, no eclipses are observed for $\phi = 0.25$ and the number and quality of the TOAs near $\phi = 0.25$ are the same as at other orbital phases, $\phi$.

PSR J1807–2459 in NGC 6544 (Ransom et al. 2001; D’Amico et al. 2001) has the lowest mass function known for any binary pulsar, with the exception of the pulsar planetary system B1257+12 (Wolszczan & Frail 1992; Wolszczan 1994). The minimum companion mass is about 0.009 $M_\odot$. This system does not display any eclipses. The most recent VLMBP discoveries, M5 C and M71 A, are the first VLMBPs in globular clusters other than 47 Tuc to show any eclipses (Ransom et al. 2002).

The mass function of a binary pulsar with a light companion is given, in solar masses, by

$$f = \frac{(c x)^3}{G M_\odot} \left( \frac{2\pi}{P_b} \right)^2 = \frac{(m_c \sin i)^3}{(m_p + m_c)^2} \simeq \frac{(m_c \sin i)^3}{m_p^2}. \quad (4)$$

Assuming that $m_p$ is similar for all pulsars (Thorsett & Chakrabarty 1999), the main cause of the differences between the mass functions of the VLMBPs should be due to differences in the companion masses and differences in system inclinations. In Fig. 7 it can be seen that all of the eclipsing VLMBPs have higher mass functions than all of the non-eclipsing VLMBPs. These are separated in the figure by a dashed line. A correlation between small mass function and the lack of eclipses can be understood if the
Figure 7. Mass function plotted against orbital period for all the binary pulsars known (circles) within the given range of orbital period and mass function. Those binaries in globular clusters are indicated inside four-pointed stars and named; we highlight those in 47 Tuc with a ‘+’. A black dot inside the symbol indicates a binary pulsar that is known to eclipse at some radio frequency; the two eclipsing Galactic binaries are also named. Binaries with $f < 3 \times 10^{-5}$ M$_\odot$ (VLMBPs) have companions with masses of 0.01–0.03 M$_\odot$, and generally present shorter orbital periods than the LMBPs ($f > 10^{-4}$ M$_\odot$). About two-thirds of the VLMBPs display eclipses. The dashed line separates the eclipsing VLMBPs from those that do not display eclipses. The inclined lines indicate constant projected semimajor axis $x$.

Companion masses are all close to 0.03 M$_\odot$. The inclination is the main factor determining the observed mass function for these objects; at high inclinations, the system is seen edge-on, eclipses can be seen and the minimum companion mass is near 0.03 M$_\odot$. At lower inclinations, no eclipses can be seen, as we are looking at the system face-on; and the mass function is lower by a $(\sin i)^3$ factor.

This hypothesis is relevant for the interpretation of the 47 Tuc J and O binary systems. If properties of eclipses are strongly determined by the inclination of the system, as for instance suggested for PSR J2051−0827 (Stappers et al. 2001; Khechinashvili, Melikidze & Gil 2000), then we can suppose that the amount of extra plasma observed near $\phi = 0.25$ of 47 Tuc J should be smaller than that observed for PSR J2051−0827 because of the former’s lower inclination (deduced from its lower mass function). We will see in the following section that this is indeed the case. A comparison with PSR B1957+20, which presents much deeper eclipses than either 47 Tuc J or PSR J2051−0827 (Fruchter et al. 1990; Ryba & Taylor 1991) despite being closer to the ‘threshold’ mass function, is not particularly meaningful, given the large difference in orbital period; we do not know if the threshold mass function, if it exists, depends in any way on the orbital period.

It is possible that the above interpretation is incorrect and that companions with a minimum mass of $\sim 0.01$ M$_\odot$ are fundamentally different in nature than those with $\sim 0.03$ M$_\odot$, being for some reason incapable of producing any sizable atmospheres. We find such an explanation unlikely, as the lighter companions must have lower surface gravities and escape velocities; it should be easier, using the same energy input from the pulsar, to create an extended atmosphere around these.

7 ECLIPSE PROPERTIES OF TWO VLMBPS

7.1 Variation of DM with orbital phase for 47 Tuc J

A previous study (Robinson et al. 1995) has suggested that 47 Tuc J shows a strongly asymmetric eclipse at 430 and 660 MHz for $0.18 < \phi < 0.40$ (we define superior conjunction as $\phi = 0.25$). Our own analysis of the data confirms this result for the 430-MHz data. For a total of 57 observations at 430 MHz, 23 include the superior conjunction. In none of these is pulsed emission detectable near the ‘eclipse phase’. We present the best successful observation in Fig. 8, which illustrates this point.
We have detected the pulsar in several high-resolution 660-MHz observations made in 1998 January and we find that the eclipses are only partial and are quite variable. A partial eclipse occurred in the orbit observed on 1998 January 11, where the strength of the signal diminished greatly during superior conjunction (see Fig. 8). This effect could possibly also be due to scintillation, which causes variations of the flux densities on time-scales of minutes to hours. However, no eclipse (partial or otherwise) was detectable in the orbit observed on 1998 January 12, only a slight delay in the arrival time of the pulses (Fig. 8). These findings suggest that the amount of plasma between the Earth and the pulsar at $\phi = 0.25$ changes continuously, on a time-scale of a day or less.

Contrary to what happens at lower frequencies, 47 Tuc J is clearly detectable at all orbital phases at 1400 MHz (Fig. 8) for the more than 200 observations made at this frequency where the pulsar is clearly detected. We also find that the DM varies with the orbital phase of the system. We determined this variation by obtaining TOAs every 8.5 min for four different subbands across the detected portion of the band. We then divide all TOAs into 18 bins of orbital phase, comprising 20' each. For the TOAs in each bin, we fit for DM using TEMPO. The result of this fitting yields an average value of DM for each orbital phase bin.

A first set of DMs was determined for the 1997–1999 data, using both the 1400-MHz TOAs (obtained with a $2 \times 96 \times 3$ MHz filter bank from 1997 August to 1999 August, with the 288-MHz band divided into four 72-MHz subbands) and the 660-MHz TOAs obtained in 1998 January and February, which were obtained using the whole 32-MHz band. The timing ephemeris used was that published in Paper I, which provides a good description of the TOAs involved in the fit. The DMs result mostly from a comparison of the 660-MHz TOAs obtained during three favourable occasions in 1998 January with the orbital ephemeris derived from the low-resolution 1400-MHz data obtained at other epochs. There is also a contribution to the determination of the DMs from the differences in pulse TOA for the different 1400-MHz subbands.

The absolute values of DMs are not very precise because it is difficult to align precisely the different standard pulse profiles at 660 and 1400 MHz (see, for example, Paper I). However, changes in DMs can be measured more accurately. We also note that, at $\phi = 0.25$, the partial eclipse causes a dearth of reliable 660-MHz TOAs. Therefore, the value of DM for that phase depends mostly on 1400-MHz data, which by itself does not have the precision to measure small changes in DM as compared to other orbital phases when both frequencies are available.

A second set of DMs was determined using only the high-resolution timing data obtained since 1999 August with the $2 \times 512 \times 0.5$ MHz filter bank. For the best observations, we divided the 256-MHz band into four 64-MHz subbands, and calculated TOAs for each subband every 8.5 min. The particular timing ephemeris used for this fit was derived using all of the 1400-MHz TOAs, from 1997 to 2002. The accuracy of the TOAs is high enough to permit the measurement of DM variations without the need for data at other frequencies. In this case, the absolute values of DM are more accurate because the problems in aligning different pulse profiles at different frequencies are minimized. They also represent an average of the variation of DM with orbital phase over many more occasions than the previous data set.

The DM data are plotted in Fig. 9. This figure shows an increase in DM at $\phi \approx 0.25$, which is clearly visible in both data sets. In this figure, we also present weighted Gaussian fits to both sets of DMs as a function of orbital phase (there is no particular theoretical argument favouring this function over any other). The parameters
be larger than 0.7 $R^\circ$ where companion. The only source of uncertainty is the inclination of the object. This is a clear indication that the companion is losing mass.

The value for $a = 0.0179$, $q = 0.0155$, $a = 1.14 R^\circ$, and $R_L = 0.13 R^\circ$ and $R_L = 0.14 R^\circ$, therefore, the Roche Lobe is much smaller than the diameter of the plasma cloud; the matter responsible for the increased DMs is not bound to the companion object. This is a clear indication that the companion is losing mass.

Excluding the TOAs with $0 < \phi < 0.5$ from the four-subband high-resolution 1400-MHz data, we obtain with TEMPO a new DM value for 47 Tuc J ($24.5848(9)$ cm$^{-3}$ pc), which is consistent with both values presented in Fig. 9. For $d(DM)/dr$, we obtain an upper limit of $2 \times 10^{-4}$ cm$^{-3}$ pc yr$^{-1}$. Although based on essentially the same multifrequency data as used by Freire et al. (2001b) – here we have included some strong detections of the pulsar in observations taken since 2001 February – this DM is, as expected, slightly smaller than that published in Freire et al. (2001b), because of the aforementioned exclusion of TOAs near $\phi = 0.25$.

7.2 Time variability of the companion’s plasma envelope

To measure all the remaining timing parameters of 47 Tuc J in Table 2, we have calculated DM corrections for every TOA; these depend on the orbital phase according to the relation presented in Fig. 9 by the middle solid line. These corrections are then taken into account by TEMPO to derive the ephemeris.

While it is not possible to directly measure a sufficiently accurate DM from individual TOAs, we can use the delays in pulse arrival times relative to the ephemeris derived above, with no DM corrections, to estimate the DM variations as a function of orbital phase, DM($\phi$). For the best observations at 660 and 1390 MHz, the residuals can be seen in Fig. 10. For the seven best 1390-MHz observations that cover any superior conjunction, the residuals can be seen in Fig. 11. These figures indicate clearly that not only DM($\phi$) changes from orbit to orbit (although keeping a maximum delay at 1400 MHz of about 0.1 $\mu$s at $\phi \sim 0.25$, which roughly corresponds to an extra DM of $0.006$ cm$^{-3}$ pc), but also that on one occasion (1999 October 11) there are apparent extra delays measured at other orbital phases, which are detected as late as $\phi = 0.5$ (indicated with an arrow in Fig. 10). This indicates a cometary-like phenomenon, with the delays at the latter orbital phases caused by material at a considerable distance from the companion. The dip in extra delay at $\phi = 0.35$ indicates that the flow of matter from the companion does not proceed smoothly.

7.3 47 Tuc O

PSR J0024–7204O is one of the four eclipsing binary pulsars discovered in the 1400-MHz survey of 47 Tuc (Camilo et al. 2000), and the only one of these for which a timing solution is known. For this pulsar, the eclipse region occupies a 50°–wide region around $\phi = 0.25$. The orbit does not have a measurable eccentricity if we exclude the TOAs with $0.1 < \phi < 0.4$. With such a circular ephemeris, we can use the same procedure used in Section 7.2 to infer the extra column densities as a function of $\phi$; simply calculate the residuals for all orbital phases, and infer the extra column density from the delays of the TOAs. It is impossible to resort to a more direct measurement of DMS such as that made for 47 Tuc J in Section 7.1, as this pulsar has an estimated 1400-MHzux density of $1.3 \times 10^6$ cm$^{-2}$, which is about 10 times smaller than the extra electron column density observed for PSR J2051–0827 at $\phi = 0.25$ (Stappers et al. 2001). Considering the separation between the pulsar and its companion for inclinations near 90°, $a = 1.14 R^\circ$, and the length of the eclipse at 430 MHz, the radius of the eclipsing object must be larger than 0.7 $R^\circ$. This implies an average electron density of $\sim 10^5$ cm$^{-3}$ near the companion.

The Roche lobe for the companion is given by (Eggleton 1983)

$$R_L = \frac{0.49 \times a q^{2/3}}{0.6q^{1/3} + \ln(1 + q^{1/3})}$$

where $q = m_p/m_c$ and $a$ is the separation between the pulsar and the companion. The only source of uncertainty is the inclination of the system. For $i = 90°$, $m_p = 0.0209 M^\odot$, $q = 0.0155$, $a = 1.14 R^\odot$, and $R_L = 0.13 R^\odot$. For $i = 60°$, $m_p = 0.0241 M^\odot$, $q = 0.0179$, $a = 1.14 R^\odot$, and $R_L = 0.14 R^\odot$, therefore, the Roche Lobe is much smaller than the diameter of the plasma cloud; the matter responsible for the increased DMs is not bound to the companion object. This is a clear indication that the companion is losing mass.

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Figure 10. Residuals as a function of orbital phase for the two best observations of 47 Tuc J at 660 MHz (filled circles) and 1390 MHz, respectively. For the later observation, the 256-MHz band was divided into four contiguous 64-MHz subbands, centred at 1294 MHz (open circles), 1358 MHz (squares), 1422 MHz (triangles) and 1486 MHz (diamonds). The TOAs and respective uncertainties were obtained with the software TREDUCE. The uncertainties were multiplied by an empirically determined factor (3.9 and 1.4 for the 660-MHz and 1390-MHz data, respectively) so as to result in a solution with a reduced $\chi^2$ of 1. A ‘tail’ of gas can be seen after superior conjunction in the observation made in the 1999 October 11.

Figure 11. Residuals as a function of orbital phase for a total of nine superior conjunctions, which appear in seven of the best detections of 47 Tuc J at 1390 MHz. The residuals were obtained with respect to the ephemeris listed in Table 2. The DM profile as a function of orbital phase clearly changes with time. The TOA uncertainties used to derive the best solution are given by the vertical line segments.

is undergoing apparently random variations in orbital period with magnitude $\Delta P_b/P_b \sim 10^{-7}$ on time-scales of about 5 yr (Arzoumanian, Fruchter & Taylor 1994; Nice, Arzoumanian & Thorsett 2000). For PSR J2051−0827, Doroshenko et al. (2001) obtain $\dot{P}_b = -16 \times 10^{-12}$, $\ddot{P}_b = 2 \times 10^{-20}$ s$^{-1}$ and $\dddot{P}_b/dt = 4 \times 10^{-28}$ s$^{-2}$, implying variations in $P_b$ similar in magnitude and time-scale to those of PSR B1957+20. We now look for similar effects in the three VLMBPs in 47 Tuc with known timing solutions: 47 Tuc I, J and O.

8 LONG-TERM ORBITAL EVOLUTION FOR 47 TUC J AND O

The eclipsing VLMBPs known in the disc of the Galaxy exhibit measurable variability in their orbital periods. PSR B1957+20

a dispersive nature, it represents an extra column density of $\sim 7 \times 10^{16}$ cm$^{-2}$.

47 Tuc O and its companion are separated by 1.2 R$_\odot$. The size of the eclipse region (0.13 of a full orbit) implies for the eclipsing region a radius larger than 0.5 R$_\odot$. This is larger than the companion’s Roche lobe (0.15 R$_\odot$ for $i = 90^\circ$), which implies again that the companion is losing mass, with significant quantities of material no longer bound to it.

Because of the extra delays indicated by Fig. 12, the TOAs with $0.1 < \phi < 0.4$ have not been considered in deriving the timing solution shown in Table 2. The DM derived after such exclusion is 24.363(10) cm$^{-3}$ pc, consistent with the value and uncertainty derived in Freire et al. (2001b).

For 47 Tuc I we obtain $\dot{P}_b = -0.4(7.2) \times 10^{-12}$ and $\dot{x} = -0.12(10) \times 10^{-12}$ (as throughout this paper, the uncertainties in this section and associated figures are indicated to 1$\sigma$, while the upper limits are indicated to 3$\sigma$). We will not discuss this pulsar any further; its limits are not constraining enough for a useful comparison with the Galactic eclipsing binary systems.

For 47 Tuc J, we can measure significant orbital evolution $\dot{P}_b = (-0.52 \pm 0.13) \times 10^{-12}$ and $\dot{x} = (-2.7 \pm 0.7) \times 10^{-14}$.

For 47 Tuc O, we can measure even stronger orbital evolution:
Figure 12. Top plots: Intensity as a function of orbital and rotational phases for three of the best observations of 47 Tuc O. The darkness is linearly proportional to the measured intensity. The residuals for the corresponding TOAs (used to obtain the best fit) are displayed in the bottom plot.

Figure 13. Variation of $P_b$ as a function of epoch for 47 Tuc O. The dashed line represents the prediction of $P_b$ based on the general TEMPO fit.

$P_0 = (9 \pm 1) \times 10^{-12}$, $\dot{P}_b = (24 \pm 8) \times 10^{-20}$ s$^{-1}$, and $|\dot{x}| < 1.8 \times 10^{-13}$. This global TEMPO fit can be compared with local fits. To measure variations in $P_b$, we divided the data for 47 Tuc O into three sets: low-resolution 1400-MHz data, MJD 50683–51458; and high-resolution 1400-MHz data, MJD 51407–51881 and MJD 51951–52357. For each data set, the orbital parameters $P_b$ and $T_{asc}$ were estimated using TEMPO. We assumed other parameters in Table 2 to hold and no variation in time for the orbital parameters. The result of the fits can be seen in Fig. 13, where the dashed line represents the global TEMPO fit. The observed variation amounts to $\Delta P_b / P_b \sim 1.1 \times 10^{-7}$.

This is very similar, both in amplitude and time-scale, to the variation in $P_b$ detected so far for the two Galactic VLMBPs. We now investigate the possible causes of this effect.

8.2 Causes for the observed variation in orbital period

The possible causes of a binary’s orbital variations are discussed in Doroshenko et al. (2001). These are: (a) energy loss due to emission of gravitational waves; (b) changing Doppler shift caused by local interstellar gravitational field; (c) mass loss from the system; (d) tidal torques; and (e) spin–orbit coupling. For PSR J2051–0827 they conclude that only the last term gives a significant contribution to the observed variations in $P_b$, as is true for PSR B1957+20 (Nice et al. 2000).

We have analysed these contributions for 47 Tuc J and O, and conclude that the only term of the total $\dot{P}_b$ that is significantly different from the case of the Galactic VLMBPs is due to the acceleration along the line of sight. This is much larger for the pulsars in 47 Tuc, because of the cluster’s gravitational field. The maximum accelerations due to the cluster along the lines of sight to 47 Tuc J and O are $a_{\text{max}} = 5.0 \times 10^{-7}$ and $1.7 \times 10^{-6}$ cm s$^{-2}$, respectively (see Paper I for details). If the binaries experienced this acceleration and no intrinsic $\dot{P}_b$, we should observe $\dot{P}_b = (a_{\text{max}}/c)P_b = -1.7 \times 10^{-13}$ for 47 Tuc J and $-6.5 \times 10^{-13}$ for 47 Tuc O. The effect should clearly be negative, because these two pulsars have negative $\ddot{P}$, and therefore a negative line-of-sight acceleration.

These contributions for the $\dot{P}_b$ of the 47 Tuc binaries are below the present detectability limits, and much smaller than the $\dot{P}_b$ observed for 47 Tuc J and O. Therefore, the only plausible cause for the observed $\dot{P}_b$ is spin–orbit coupling, as for the two Galactic VLMBPs. This effect may be due to distortions of the companion caused by its magnetic field, discussed in detail by Applegate & Shaham (1994).
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The $P_t$ measured for PSR J2051−0827 is 30 times larger than the present value for 47 Tuc J. We believe this finding is not particularly significant. The long-term timing for PSR B1957+20 and the higher derivatives of the orbital period measured for 47 Tuc O and PSR J2051−0827 show that the $P_t$ for these systems is constantly changing in magnitude and sign; at some epochs this quantity is small, as is currently the case for 47 Tuc J. However, 47 Tuc J has a measured $\dot{\omega}$ eight times smaller than for PSR J2051−0827, which suggests that spin–orbit coupling – the only viable cause of the observed $\dot{\chi}$ of PSR J2051−0827 (Doroshenko et al. 2001) – might indeed be weaker for 47 Tuc J.

A conclusive detection of the VLMBPs companions in 47 Tuc at optical wavelengths would in principle give us important clues about their natures. If these objects have the same absolute magnitude as the companion of PSR B1957+20 ($R = 19.5$ at $D = 1.6$ kpc; Callanan, van Paradijs & Regelink 1995) or PSR J2051−0827 ($R = 22.3$ at $D = 1.3$ kpc; Stappers, Bessell & Bailes 1996) then their magnitude at 47 Tuc would be 21.9 and 25.2, respectively. At least the first should be detectable in the deepest HST data set (Gilliland et al. 2000).

9 SUMMARY

We have improved the time-resolution of the observations of the pulsars in 47 Tuc by a factor of 3, leading immediately to much improved pulse profiles. The results presented in this paper illustrate the variety of goals that can be achieved through pulsar timing over a long time baseline with improved levels of precision.

We have now measured proper motions for 11 pulsars. We have, for the first time, detected intrinsic motions of the pulsars relative to each other; the proper motion of 47 Tuc D is inconsistent with the motion of 47 Tuc E and J. We have introduced tighter constraints to the cluster parameters using the newly derived period derivative of 47 Tuc S. We have also measured a significant second period derivative for 47 Tuc H; this cannot be caused by a variation of the gravitational field of the cluster at the pulsar’s changing location.

The improved measurement of $\omega$ for this pulsar implies a total mass for the binary of $(1.61 \pm 0.04) M_\odot$, assuming (reasonably) that $\dot{\omega}$ is entirely due to general relativity. We have also introduced severe constraints for the mass of any planet orbiting any of the pulsars in 47 Tuc – no planets can be clearly detected.

We have noted that all of the eclipsing VLMBPs have higher mass functions than all of the non-eclipsing VLMBPs. This suggests that the main determinant of their mass functions is the inclination of the binaries, but it is also possible that the lower-mass systems are of a fundamentally different nature. We have measured the average DM variation with orbital phase for 47 Tuc J; the column density at 47 Tuc would be 21.9 and 25.2, respectively. At least $\Phi_0$ and $\dot{\chi}$. 

APPENDIX

We here describe how we computed the Lomb–Scargle spectra for the pulsars in 47 Tuc (Section 4.3). This is an improved treatment compared to the numerical relations presented in Bell et al. (1997).

Starting with the definition of the mass function, we have

\[ m_c \sin i = a_p \sin i \left( \frac{4\pi^2 m_p^3}{GP_b^2} \right)^{1/3}. \]  

where \( m_p \) and \( m_c \) are the pulsar and companion masses, \( i \) is the inclination of the orbit with respect to the plane of the sky, \( P_b \) is the orbital period, \( G \) is the universal gravitational constant and \( a_p \) is the semimajor axis of the pulsar’s orbit about the common centre of mass. Assuming a pulsar mass of \( m_p = 1.4 \, M_\odot \) we obtain

\[ m_c \sin i = 1.42 \times 10^{-4} a_p \sin i P_b^{-2/3}. \]  

with \( P_b \) expressed in days, \( a_p \sin i \) in metres and \( m_c \sin i \) in Earth masses.

The Lomb–Scargle periodograms of \( N \) timing residuals (obtained after fitting for the best timing model, presented in Tables 1 and 2) are computed using an algorithm presented by Press et al. (1992). The result is a distribution of power as a function of orbital frequency, \( \mathcal{P}(\nu) \equiv 1/P_b \). As shown by Scargle (1982), an unmodelled harmonic signal present in the timing residuals of the form

\[ x(t) = x_0 \sin(2\pi/P_b t + \phi_0) \]  

will cause a signal in the power spectrum of magnitude

\[ \mathcal{P}(\nu) = N \left( \frac{x_0}{2} \right)^2. \]  

Note that \( N \) (the number of timing residuals) is usually much smaller than the number of frequency points in the periodograms.

From the power spectrum \( \mathcal{P}(\nu) \), we can infer an upper limit for the amplitude of any possibly existing harmonic signal. Using equation (A4), we find

\[ x_0 \leq 2 \sqrt{\frac{\mathcal{P}(\nu)}{N}}. \]  

We can identify the amplitude \( x_0 \) of a signal hidden in the TOAs as the light travel time across the orbit, i.e. \( a_p \sin i = c x_0 \). Using equation (A2), we obtain

\[ m_c \sin i \leq 1.42 \times 10^{-4} x_0 P_b^{-2/3}. \]  

As the power in the spectrum scales with \( x_0^2 \) it is essential to use consistent units for this back transformation from spectral values into the time domain. Computing spectra from timing residuals measured in microseconds, we finally obtain with equation (A5)

\[ m_c \sin i \leq 0.0852 \sqrt{\frac{\mathcal{P}(\nu)}{N}} P_b^{-2/3}. \]  

where units of \( m_c \) and \( P_b \) are again Earth masses and days, respectively. Wenote that the numerical factor in this equation differs from that presented by Bell et al. (1997), who do not specify the units of their timing residuals.

Finally, we use equation (A7) to convert our computed Lomb–Scargle periodograms into diagrams presenting upper limits of an unknown companion mass as a function of orbital period. During this procedure, we follow the arguments of Bell et al. (1997) and multiply periods of 1 yr or longer with \( P_b \) to account for a strong covariance between the orbital period of a planet and the period derivative of the pulsar (see Thorsett & Phillips 1992).

In contrast to Bell et al. (1997), we compute confidence limits using Monte Carlo simulations where we create random sets of timing residuals with identical sampling, mean and variance as the original data set. For each random data set, we compute the Lomb–Scargle periodogram and record the maximum amplitude, \( z \), for each run. The cumulative probability function (Scargle 1982) follows

\[ p = (1 - \exp(-z))^M, \]  

where \( M \) is a number of the order of the points in the periodogram. We determine each \( M \) by a least-squares method comparing equation (A8) with the observed cumulative distribution. The amplitude \( z_0 \) of a random signal to occur with a probability of \( p_0 \) can then be determined from

\[ z_0 = -\ln \left[ 1 - (1 - p_0)^{1/M} \right]. \]  

These are the upper lines shown in Fig. 5 and on the remaining Lomb–Scargle plots in http://www.jb.man.ac.uk/~pulsar/47Tuc.html.

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