Problem set #3   Biology 463
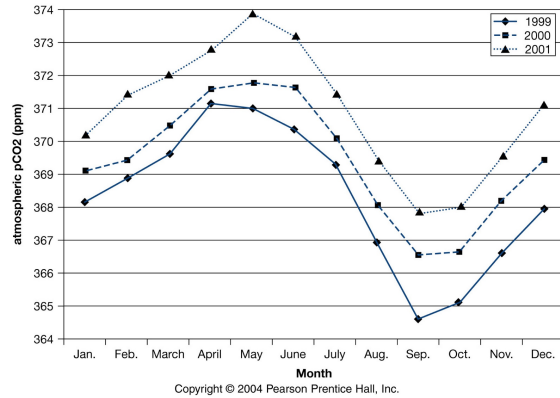
Working individually, correctly answer the following questions. To get credit for your answer, you **must show all work in a legible and organized way.**

Copyright © 2004 Pearson Prentice Hall, Inc.

Seasonal fluctuations in atmospheric CO2 from the Mauna Loa Observatory for 1999-2001. The gradual increase due to fossil-fuel burning and deforestation accounts for the offset from year to year.

1) Use the graph shown above to answer the following questions:

During which seasons in 1999 (summer = June – Aug.; fall = Sept. – Oct.; winter = Nov. – Mar.; spring = Apr. - May) is the rate of photosynthesis <u>greatest</u>, relative to the combined rate of respiration and decomposition, and during which seasons is it <u>smallest</u>? Explain your reasoning. Why aren't these coincident with the minimum and maximum CO_2 levels for the year, respectively? *Hint: you may find class notes & the information on pages 151-152 of your textbook to be helpful.*

2) Read pages 15-17 in Consider a Spherical Cow (see attached information), and complete Exercises 1 & 3. *Use the values for petroleum, natural gas, and coal consumption found on page 240 of the "Appendix of useful numbers from Spherical Cow" that is found on class web site.*

3) Read pages 28-29 in Consider a Spherical Cow (see attached information), and complete Exercise 1.

4) Read pages 100-102 in Consider a Spherical Cow (see attached information), and complete Exercises 2 & 3. *Note: There are $1.8x10^{20}$ moles of air in the atmosphere and there was $735x10^{12}$ Kg C as CO_2 in the atmosphere in 1980. Use the values for the mole fraction of gases in the atmosphere that are found on page 237 of the "Appendix of useful numbers from Spherical Cow" that is found on class web site.*

To receive credit your answers must be emailed to me as a PDF file **by the date indicated on the syllabus**.

6. The Greens We Eat

What fraction of the total annual plant growth on Earth was eaten by humans in 1983?

· · · · · · ·

Just for fun, take a guess and write your guess down. Guessing things like this and then later comparing your guess with the result of a quantitative derivation helps sharpen your intuition. Now let's proceed to the derivation.

The fraction we are looking for has annual human food consumption as its numerator and annual global green plant production as its denominator. You must make several choices before you can calculate this fraction.

First, choose your units. You can determine the numerator and denominator in units of heat energy (e.g., calories) or in grams of carbon, dry-weight biomass, or wet-weight biomass. Of course, if you convert properly, you should get about the same answer using any of these units.

Second, decide what is meant by annual plant growth. Will you take the green-plant production rate to be the gross primary productivity (total photosynthetic activity) or the net primary productivity (gross productivity minus losses due to plant respiration)? The answer will depend on which choice you make.

Third, be specific about the interpretation of human food consumption. Specifically, you can count meat consumption on the same caloric or weight basis as plant matter, or you can estimate how much plant matter it took to produce a unit of meat matter.

We will solve the problem using energy units and *net* primary productivity (npp). Net productivity is clearly the more sensible denominator because photosynthetic product respired by plants is of no benefit to plant eaters. We will not worry for now about whether human energy needs are met by meat or green plants. (That issue is taken up in Exercise 2.) Here we will simply compare the total food energy consumed per year to the npp in units of energy per year.

The annual energy content of the typical human diet is given in the Appendix (XV), although you can probably derive it from something you may know, namely, that the average person consumes about 2500 Calories per day.[7] Calories with a capital "C" are weight-watchers' calories. One Calorie equals 1000 physicists' calories (lower

---

7. This figure represents a rough average of the food energy consumption in the underdeveloped nations, where many people have access to fewer Calories each day, and in the overdeveloped nations, where many people eat more than 2500 Calories per day.

case "c").[8] Therefore, the average person consumes about 2.5×10^6 cal/day or 9×10^8 cal/yr. Multiplying by the present (1983) estimated world population of 4.7×10^9 people, we arrive at an annual rate of human food consumption of about 4.2×10^{18} cal/yr. In this book, and in most of the scientific literature today, the preferred energy unit is the joule (J). Using the conversion tables in the Appendix (I.8), we can express our numerator as 1.8×10^{19} J/yr.

The denominator, net primary productivity (in units of joules per year), can be obtained from information given in the Appendix (XII.1). Net primary productivity (in units of grams of carbon per year) is given as 7.5×10^{16} g(C)/yr.[9] This is the net amount of carbon converted from CO_2 to carbon-containing organic molecules each year. It includes photosynthate grazed by herbivores or detritivores subsequent to formation but does not include photosynthate that the plant itself burns for metabolic purposes. The Appendix also notes (VII.4) the energy content of dry biomass—about 1.6×10^4 J/g(biomass).

With npp in units of g(C)/yr and energy content of dry biomass in units of J/g(biomass), we can express npp in the same units as the numerator, J/yr, provided we can relate grams of dry biomass to grams of carbon. Thus, we need to know the fraction, by weight, of carbon contained in biomass, and then we can use the unit conversion formula:

$$\text{npp (J/yr)} = \frac{\text{npp [g(C)/yr]} \times \text{energy content [J/g(biomass)]}}{\text{carbon content [g(C)/g(biomass)]}}. \quad (1)$$

The carbon content of dry biomass can be estimated by looking at a typical plant compound and using its fractional carbon content as an approximation. Glucose is a common product of photosynthesis. Its chemical formula is $C_6H_{12}O_6$, meaning simply that a molecule of glucose contains 6 atoms of carbon, 12 of hydrogen, and 6 of oxygen. Exercise 1 provides a more accurate chemical representation of the atomic constituents of dry biomass, but here we can use the formula for glucose as an approximation. The molecular mass of glucose is $6\,m(C) + 12\,m(H) + 6\,m(O)$, where $m(X)$ is the atomic mass of element X. Thus the molecular mass of glucose is $6(12) + 12(1) + 6(16) = 180$ (see Figure I-2). The molecular mass of the 6 carbon atoms in glucose is $6\,m(C) = 72$, from which we deduce that

$$\text{carbon content} = \frac{g(C)}{g(\text{biomass})} = \frac{72}{180} = 0.4 \quad (2)$$

---

8. The physicists' calorie denotes the amount of heat needed to raise a gram of water from 14.5°C to 15.5°C.

9. The Appendix (I.1–14) also lists the unit abbreviations (e.g., g for grams) used throughout the book.
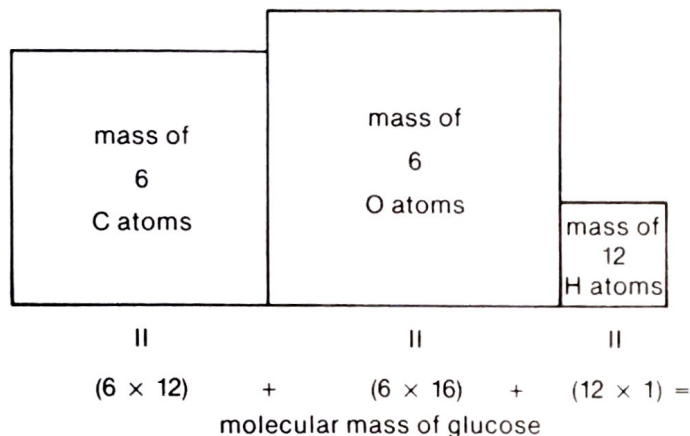
Figure I-2 The molecular mass of glucose ($C_6H_{12}O_6$) is calculated by adding together the mass of 6 carbon atoms (6×12), 6 oxygen atoms (6×16), and 12 hydrogen atoms (12×1).

Substituting Eq. 2 into Eq. 1, and using the above data from the Appendix,

$$npp(J/yr) = \frac{7.5 \times 10^{16} \text{ g(C)/yr} \times 1.6 \times 10^4 \text{ J/g(biomass)}}{0.4 \text{ g(C)/g(biomass)}} \tag{3}$$

$$= 3.0 \times 10^{21} \text{ J/yr.}$$

Putting all of the above together, the fraction, f, of npp consumed by humans is given by

$$f = \frac{\text{rate of human food consumption (J/yr)}}{npp \text{ (J/yr)}}$$

$$= \frac{1.8 \times 10^{19} \text{ J/yr}}{3.0 \times 10^{21} \text{ J/yr}} \tag{4}$$

$$= 0.006.$$

In words, the rate at which energy is consumed by humans as food is about 0.6% or $1/160$ of the net rate at which energy is incorporated as plant matter in photosynthesis. How close was your guess?

✳ *EXERCISE 1:*   A formula representing the approximate chemical composition of typical dry freshly photosynthesized biomass is H_{2960} O_{1480} C_{1480} N_{160} P_{18} S_{10}, where each subscript denotes the relative number of atoms of that elemental type. If this more precise representation is used instead of $C_6H_{12}O_6$, recalculate the fraction, f.

EXERCISE 2:   The production of animal-derived foods, such as beef, eggs, fish, and milk, requires the production of plants as fodder.

To produce 1 J of energy in the form of beef requires about 8 J of energy in the form of grains, while for poultry about 3 J of energy from grains are required. These represent extremes. The production of 1 J of other animal-derived foods requires very roughly 5 J of plant matter. Estimate how much meat you eat per year and use this to work out the following: If all of Earth's people ate a diet like yours, approximately what would the fraction f be? (Hint: if you start with an estimate of the mass of meat you eat, you will have to assume something about its water content. You may assume fresh meat has about the same water content as fresh vegetation—roughly 70%.)

✳ *EXERCISE 3:* About what fraction of Earth's current npp would we need to consume if we derived all the energy we now (1980) get from fossil fuel from biomass instead? What does your answer tell you about the wisdom of replacing all fossil fuels with biomass? What ecological problems would you anticipate this might cause?

EXERCISE 4: If the human population continues to grow at about 2%/yr, in what year will humans be eating at Earth's current rate of npp?

3. Carbon in the Biosphere

What are the residence times of carbon in continental and marine vegetation?

. . . .

Take a guess. Trees live for 50 years or more and constitute a large share of continental vegetation. Phytoplankton, accounting for a large share of marine primary productivity, generally come and go in a series of annual blooms and crashes each lasting less than a year. We might therefore expect the residence time of carbon in plants to be on the order of 10–100 years on the land and 0.1–1 year in the oceans.

To be more accurate, we must know the stocks and flows of carbon. The stocks are living biomasses and the flows are net primary productivities. The Appendix provides both of these quantities, in conveniently comparable units—g(C) and g(C)/yr. From this we can obtain

$$T_{terrestrial} = \frac{\text{stock of living continental biomass}}{\text{continental net primary productivity}}$$

$$= \frac{5.6 \times 10^{17}\ g(C)}{5 \times 10^{16}\ g(C)/yr} \tag{1}$$

$$= 11.2\ yr$$

and

$$T_{oceanic} = \frac{\text{stock of living marine plants}}{\text{marine net primary productivity}}$$

$$= \frac{2 \times 10^{15}\ g(C)}{2.5 \times 10^{16}\ g(C)/yr} \tag{2}$$

$$= 0.08\ yr \approx 1\ \text{month.}$$

These relations are illustrated in Figure II-2. The considerable uncertainty in the stock and flow data used here (see Appendix) makes these residence times approximate.

Notice that the continental residence time of carbon is considerably shorter than the average lifetime of trees. Even though the woody parts of trees constitute the bulk of living continental biomass, only a part of each tree's annual production adds woody tissue; much of the net primary productivity produces leaves, which have less than a one-year residence time.

Why did we use net primary productivities in this calculation? Had we used data on gross marine and continental primary productivities

Terrestrial biomass
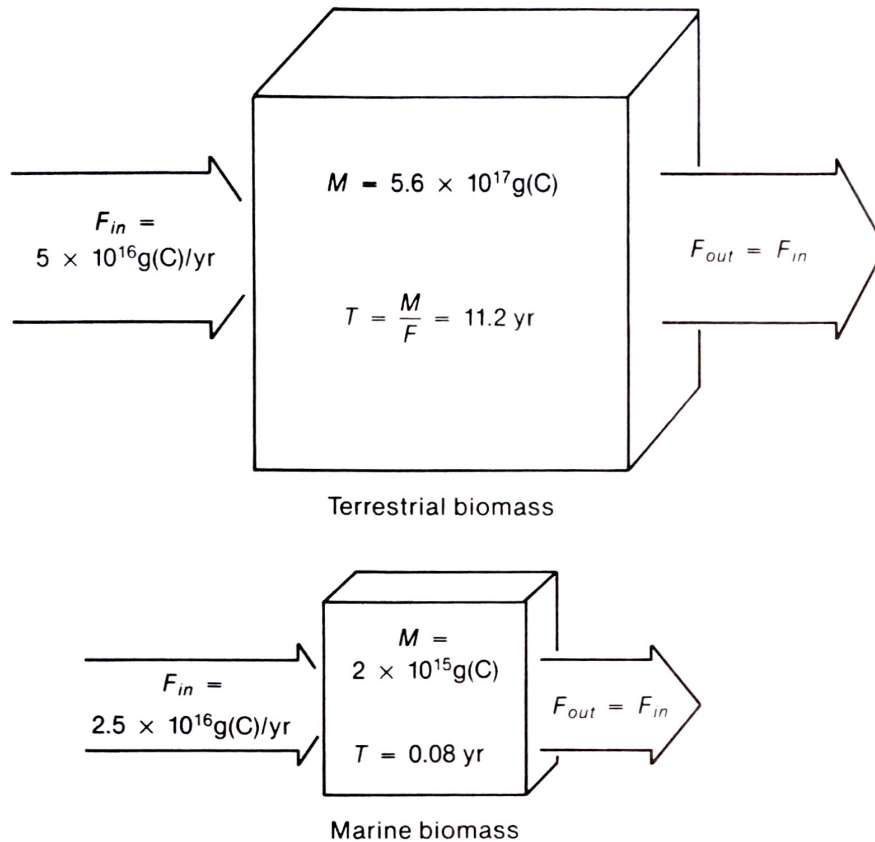
Marine biomass

Figure II-2 Stocks and flows of biomass in terrestrial and marine biomasses.

instead, we would have obtained much shorter residence times. Carbon flows relatively quickly through respiratory pathways in organisms. This respiratory flow is not included in net primary productivity. Using net primary productivity yields a residence time that bears a closer relation to the lifetimes of typical organisms (see Exercise 2).

EXERCISE 1: Suppose that the average residence time (ignoring respiratory pathways) of carbon in the phytoplankton in a lake is two weeks. Zooplankton in the lake, grazing upon the phytoplankton, consume 40% of the net primary productivity and have an incorporation efficiency of 25% (i. e., 25% of the phytoplankton biomass they eat is incorporated into zooplankton biomass). In other words, the net productivity of the zooplankton is 0.25 × 0.40 or 10% of the npp of the algae. If the average residence time (ignoring respiratory pathways) of carbon in zooplankton biomass is six months, estimate the ratio of the average biomass of the zooplankton population to that of the phytoplankton population in the lake. Figure II-3 illustrates the flows in and out of the two stocks of plankton.

* *EXERCISE 2:* In this exercise we explore the relation between residence time of biomass and lifetime of individuals in a steady-state
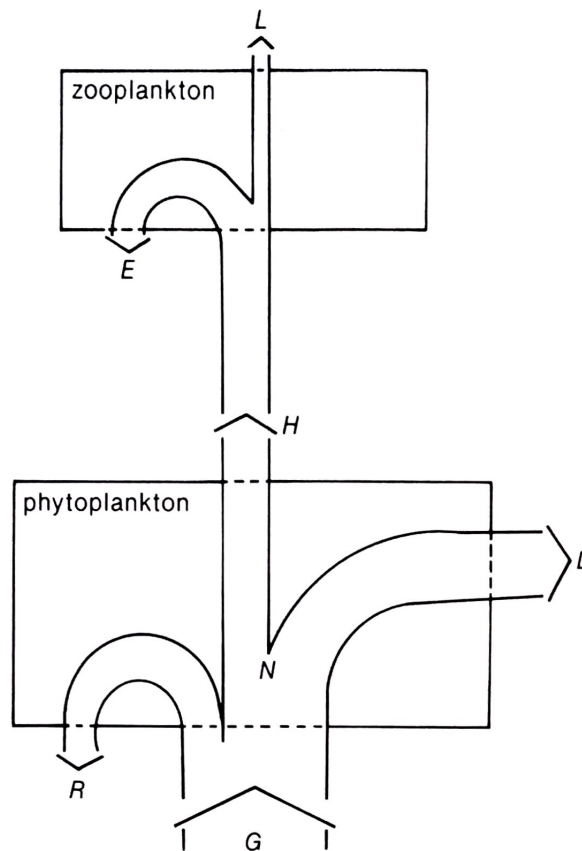
Figure II-3 The flows of carbon in and out of a phytoplankton and zooplankton population in a lake. G is the gross primary productivity of the phytoplankton and R is their respiration rate. $N = G - R$ is net primary productivity. H represents loss of carbon by zooplankton herbivory, and D comprises all other losses. E is excretory and metabolic loss of carbon from the zooplankton population, and $L = H - E$ describes the loss of carbon from the zooplankton population by predation or other forms of death. N and L are the flows used to determine the carbon residence time in the biomass of each population.

population. The residence time, T, of biomass in a population is the interval during which new net production of biomass equals the total average standing crop or stock, M. To relate T to lifetime of individuals, we must know something about the mass-versus-age dependence of the population. Consider two cases: (a) All individuals grow linearly in mass from birth until death. (b) All individuals achieve maximum mass early in life and then maintain a constant mass until death. The first case corresponds very crudely to the situation among plant populations, the second to that among animals. If it is assumed that all individuals in a population die at the same age, A_0, show that in case a, T equals $A_0/2$ and in case b, T is about equal to A_0.

* *EXERCISE 3:* Consider a forest in which the residence time for carbon (ignoring respiratory pathways) in the living trees is 15 years,

19. Altering the Atmosphere by Burning Fossil Fuels

In 1980, how much O_2 was removed from the atmosphere due to the combustion of fossil fuel on Earth, and how much CO_2 and H_2O were produced in the combustion process?

· · · · · · ·

The primary elemental constituents of the three major types of fossil fuel (natural gas, petroleum, and coal) are carbon and hydrogen. When fossil fuel is burned, oxygen from the atmosphere combines with the carbon to make CO_2 and with the hydrogen to make H_2O. In addition, coal contains some water (typically 10–15% by weight), which is released to the atmosphere upon combustion of the coal. All three types of fossil fuel contain various other substances such as ash, sulfur, and trace metals in even lower concentrations.

The Appendix (VII.2–4) provides detailed information about the average chemical composition of the three fossil fuels, the heat value of a unit amount of each fuel, and the total amount of heat energy derived worldwide from each in 1980. To use this information to compute the rates of CO_2 and H_2O emission and of oxygen consumption, we need to balance the chemical combustion reactions for each fuel. Consider petroleum, first, with the approximate chemical composition $CH_{1.5}$. The combustion reaction for this fuel is

$$CH_{1.5} + xO_2 \rightarrow yCO_2 + zH_2O, \tag{1}$$

where x, y, and z are called stoichiometric constants. Their values, to be determined below, provide the answer to our problem: For each mole of petroleum consumed, x moles of oxygen are consumed, and y and z moles, respectively, of CO_2 and H_2O are produced. The number of moles of petroleum burned in 1980, in turn, can be determined from the data in the Appendix (VII.2–4).

Let's first determine the stoichiometric constants. Equating moles of C, H, and O on each side of the reaction reveals constraints on x, y, and z. One mole of carbon in $CH_{1.5}$, for example, produces y moles of C in the form of CO_2, and so

$$y = 1. \tag{2}$$

Similarly, one and a half moles of H in $CH_{1.5}$ produces z moles of H_2 or $2z$ moles of H, and so

$$2z = 1.5 \tag{3}$$

or

$$z = 0.75. \tag{4}$$

Finally, $2x$ moles of O produces $2y$ moles of O in CO_2 and z moles of O in H_2O, or

$$2x = 2y + z. \tag{5}$$

Combining Eqs. 2, 4, and 5, we get

$$x = 1.375. \tag{6}$$

The number of moles of $CH_{1.5}$ burned worldwide in 1980 now has to be determined. From the Appendix (VII.2–4), we learn that the energy content of the combusted petroleum was 1.35×10^{20} J; petroleum has a heat content of 4.3×10^{10} J/tonne. By weight, petroleum is 98% $CH_{1.5}$; thus, the amount of $CH_{1.5}$ consumed in 1980 was

$$0.98 \times \frac{1.35 \times 10^{20} \text{ J}}{4.3 \times 10^{10} \text{ J/tonne}} = 3.08 \times 10^9 \text{ tonnes}(CH_{1.5})$$
$$= 3.08 \times 10^{15} \text{ g}(CH_{1.5}). \tag{7}$$

Since one mole of $CH_{1.5}$ has a mass of $12 + 1.5$ or 13.5 g, the number of moles of $CH_{1.5}$ consumed in 1980 was

$$\frac{3.08 \times 10^{15}}{13.5} = 2.28 \times 10^{14} \text{ moles}(CH_{1.5}). \tag{8}$$

Hence $n(O_2)$, the number of moles of consumed O_2, was x times this value, or

$$n(O_2) = 1.375 \times 2.28 \times 10^{14}$$
$$= 3.14 \times 10^{14} \text{ moles}(O_2). \tag{9}$$

The values of $n(CO_2)$ and $n(H_2O)$ can be obtained by multiplying 2.28×10^{14} moles($CH_{1.5}$) by y and z, respectively. Hence,

$$n(CO_2) = 2.28 \times 10^{14} \text{ moles}(CO_2) \tag{10}$$

and

$$n(H_2O) = 1.71 \times 10^{14} \text{ moles}(H_2O). \tag{11}$$

A similar procedure is used for calculating $n(O_2)$, $n(CO_2)$, and $n(H_2O)$ from coal and natural gas combustion. However, now the more diverse chemical nature of the fuel must be taken into account. Referring to the Appendix (VII), we find that the composition of natural gas, expressed as mole fractions, is CH_4 (75%), C_2H_6 (6%), C_3H_8 (4%), C_4H_{10} (2%), and C_5H_{12} (1%). The remaining 12% in noncombustible. Using these mole fractions of each hydrocarbon in natural gas, you can show that a mole of that fuel contains [0.75(1) + 0.06(2) + 0.04(3) + 0.02(4) + 0.01(5)] moles of C and [0.75(4) + 0.06(6) + 0.04(8) + 0.02(10) + 0.01(12)] moles of H. Thus, the effective formula of the combustible portion of the fuel is $C_{1.12}H_4$.

The stoichiometric constants can now be determined as before, resulting in

$$C_{1.12}H_4 + 2.12O_2 = 1.12CO_2 + 2H_2O. \tag{12}$$

The number of moles of natural gas burned in 1980 will equal the total heat value derived (6.0×10^{19} J; see Appendix, VII.2) divided by the number of joules per cubic meter of gas (3.9×10^7 J/m^3; see Appendix, VII.4), and then multiplied by the number of moles per cubic meter (44.6 moles/m^3 for any gas, since one mole of any gas occupies 22.4 liters at standard temperature and pressure). This works out to be 6.9×10^{13} moles of natural gas. Multiplying by 0.88 (the combustible fraction) and by the approximate stoichiometric constants gives the number of moles of O_2 consumed and of CO_2 and H_2O produced.

For coal, the only subtlety is that 13% of coal is water, which is liberated to the atmosphere upon combustion. This must be included in the calculation.

The final result for all three fuels is conveniently summarized in the following table (see Exercise 1).

	$n(CO_2)$*	$n(H_2O)$*	$n(O_2)$*
Petroleum	2.28	1.71	3.14
Natural gas	0.68	1.21	1.28
Coal	1.80	0.94	2.16
Total	4.76	3.86	6.58

*($\times 10^{14}$ moles)

EXERCISE 1: Derive the results for natural gas and coal shown in the table above.

✖ **EXERCISE 2:** If all the CO_2 released to the atmosphere in 1980 from fossil-fuel burning remained there, by what percentage would that year's CO_2 emission increase the atmospheric concentration?

✖ **EXERCISE 3:** By what percentage did the O_2 consumed in 1980 by fossil-fuel burning deplete the atmosphere's stock of O_2?