Problem set #2

Working individually, correctly answer the following questions. To get credit for your answer, you **must show all work in a legible and organized way.**

1. Consider a forested watershed in which evapotranspiration removes about 50% of the annual rainfall and there are no losses of water to groundwater. A scientist makes the following observations:

Precipitation: Amount: 100 cm/year

Stream water: Volume: 1.5 x 10⁸ L/year

What is the area of the watershed drained by the stream in m²?

Hint: Think about net precipitation and what it represents, and be aware that $1 L = 1,000 \text{ cm}^3$

2. Read pages 50-53 from "Consider a Spherical Cow" that are given below. Then answer the following questions:

a) By how much will the **total global** precipitation rate change?

b) Changes in hydrologic flows will result in changes in stocks and/or residence times. If the mean residence times for water in each of the major water compartment on Earth (i.e., the atmosphere, oceans, and land) are **unchanged** as a result of the changes in the flows worked out in the material you read, then deduce whether the new equilibrium stocks of water in each major compartment will have increased, decreased, or stayed the same compared to the initial stocks of water in each compartment.

3. The aggregate temperature of surface waters carried into the northern Atlantic is about 11°C. That of the aggregate deep water formed in the northern Atlantic is about 3°C. The amount of water downwelling in the north Atlantic every year is 10.28 x greater than the total flow of water in the world's runoff (which is 46x10¹² m³/yr). The specific heat of liquid water is 4.184 J/cm³/°C. Using this information, calculate the annual amount of energy released by downwelling water in the north Atlantic. Compare this value to an estimate you find online for the world's electrical energy production. Be sure to note where you found your estimate and the year the estimate is for.

Email your answers to me as a PDF file by the date indicated on the syllabus.

9. Where Would All the Water Go?

If evapotranspiration from Earth's land area were to diminish by 20% uniformly over the land area, as might result from widespread removal of vegetation, what changes would occur in the globally averaged precipitation on the land surface and in the globally averaged runoff from the land to the sea?

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Because we need not consider stocks or time constants, this is not a residence-time problem. It is a box-model problem, however, requiring careful identification of boxes and balancing of inflows and outflows for each box. An immediate guess might be that precipitation would decrease by 20%; but this would be correct only if the hydrocycle did not link our two boxes, the continents and the seas. The existence of runoff from the land to the sea implies that some evaporation from the sea falls as precipitation on the land. Because this portion of land precipitation will not be affected by the 20% decrease in evapotranspiration from the land, the overall effect will be less than 20%.

To solve the problem, a systematic look at the global water budget is helpful. The following water flow rates can be defined:

- P_L = rate of precipitation on the land
- $P_{\rm s}$ = rate of precipitation on the sea
- R = rate of runoff from the land to the sea
- E_{LL} = rate of evapotranspiration from the land that falls as precipitation on the land
- E_{LS} = rate of evapotranspiration from the land that falls as precipitation on the sea
- E_{SS} = rate of evaporation from the sea that falls as precipitation on the sea
- E_{SL} = rate of evaporation from the sea that falls as precipitation on the land

These flows are illustrated in Figure II-9.

Our problem can now be restated in terms of these definitions: How will R and P_L change if E_{LL} and E_{LS} both diminish by 20%? There are three water-conservation relations among the seven quantities we have defined. The first relation states that water is conserved in the sea:

$$P_S + R = E_{SS} + E_{SL}. \tag{1}$$



Figure II-9 The flows of water in the global hydrocycle.

The second states that water is conserved on land:¹⁰

$$P_L = R + E_{LL} + E_{LS}, \tag{2}$$

and the third states that the rate of water flow from land to sea equals the rate from sea to land:

$$R + E_{LS} = E_{SL}.$$
 (3)

Any two of these can be derived from the third plus the two identities that follow from the definitions:

$$P_L = E_{LL} + E_{SL'} \tag{4}$$

and

$$P_S = E_{SS} + E_{LS}.$$
 (5)

All told, there are three independent relations among the seven quantities. Thus, four independent empirical values are needed to determine all seven quantities.

In the Appendix (VI.3), values for P_L , P_S , and R are given. The fourth piece of information we need is that approximately 25% of the evapotranspiration from the land precipitates on the sea, while 75%,

^{10.} This equation is correct only if the stock of groundwater is unchanged. Exercise 2 asks you to examine this assumption.

or three times as much, precipitates on the land ($E_{LL} = 3E_{LS}$). (Exercise 4, below, suggests a procedure for deriving this estimate of the ratio of E_{LL} to E_{LS} . The value of 3 given here is only approximate; a precise empirical value is not available.) With this information and the equations above, we obtain the following values: $P_L = 108 \times 10^3 \text{ km}^3/\text{yr}$; $P_S = 410 \times 10^3 \text{ km}^3/\text{yr}$; $R = 46 \times 10^3 \text{ km}^3/\text{yr}$; $E_{LL} = 46.5 \times 10^3 \text{ km}^3/\text{yr}$; $E_{LS} = 15.5 \times 10^3 \text{ km}^3/\text{yr}$; $E_{SS} = 394.5 \times 10^3 \text{ km}^3/\text{yr}$; and $E_{SL} = 61.5 \times 10^3 \text{ km}^3/\text{yr}$. As you may suspect, non-significant figures have accumulated here; we can drop them later.

If the reduction in evapotranspiration is uniformly distributed over the land, it is reasonable to assume that E_{LL} and E_{LS} each decrease by 20%. (Note, on the other hand, that if the decrease in evapotranspiration occurred primarily along the coastlines of the continents, then this assumption would be a poor one.) Using primed quantities (P'_L , P'_S , R', etc.) to denote the rates subsequent to the 20% decrease in evapotranspiration, we can write

$$E'_{SS} = E_{SS},$$

$$E'_{SL} = E_{SL},$$

$$E'_{LL} = .8 E_{LL},$$
(6)

and

$$E_{LS}' = 0.8 E_{LS}$$

Then, setting up new conservation equations and identities for the primed quantities, the primed versions of Eq. 3 and 4 become:

$$R' = E'_{SL} - E'_{LS}$$
(3')

and

$$P'_{L} = E'_{LL} + E'_{SL}.$$
 (4')

Use of Eq. 6 then leads to

$$R' = E_{SL} - 0.8 E_{LS}, \tag{7}$$

$$P'_L = 0.8 E_{LL} + E_{SL}.$$
 (8)

Using Eqs. 3 and 4, these can be rewritten as

$$R' = R + 0.2 E_{LS}$$
(9)

and

$$P'_L = P_L - 0.2 E_{LL}. \tag{10}$$

Numerically, $R' = (46.0 + 3.10) \times 10^3 \text{ km}^3/\text{yr}$, which is about a 7% increase over R; and $P'_L = (108 - 9.30) \times 10^3 \text{ km}^3/\text{yr}$, which is about a 9% decrease from P_L .

Some readers will be able to solve this problem without plodding through all the formal steps presented here. It is helpful to see how to formalize problems like this one, however, because more complicated situations often make this approach necessary.

EXERCISE 1: By how much will the total global precipitation rate change?

* EXERCISE 2: Changes in hydrologic flows will result in changes in stocks and/or residence times. If the residence times for water in each of the major water compartments on Earth (atmosphere, ocean, ice, soil moisture, groundwater, lakes, and streams) are unchanged as a result of the changes in the flows worked out above, try to deduce which stocks of water will increase, which will decrease, and which will stay the same. For each compartment, would you expect residence time or stock of water to remain more nearly constant subsequent to the changes in flows?

* EXERCISE 3: A reduced rate of precipitation on the land, resulting directly from reduced transpiration, may lead to a reduction in plant growth. On the other hand, reduced transpiration may lead to more moist soil, perhaps enhancing plant growth. Either of these changes in plant growth will alter the transpiration rate again. Such a circular chain of events is called a feedback process. Our solution to the main problem, above, was deficient because it neglected feedback effects. Another feedback process that can result from a decreased evapotranspiration rate is a decreased rate of latent heat transfer from the surface upwards, thus increasing both surface temperature and evaporation rate. Removal of vegetation could induce other climate changes, including an increased CO2 greenhouse warming and an altered radiation balance caused by changes in the surface albedo (reflectance) of the devegetated land. An increased atmospheric CO₂ concentration would probably further decrease the rate at which plants transpire water.

These effects will be discussed quantitatively in subsequent problems (III.7,8), but for now try to describe the qualitative nature of these feedback effects on P'_L . An amusing wrinkle to consider is that with increased runoff the amount of water in the sea would rise, thus increasing its surface area. Would this significantly increase the rate of evaporation from the sea and thus alter our answer?

* EXERCISE 4: Using (a) the residence time for atmospheric water derived in Problem II.2 and (b) a map of the world, estimate the ratio of E_{LL} to E_{LS} .